These are the topics of the next seven sections of this chapter, after which we conclude with the creeping-flow approximation (Sec. 3-9) and digital computer solutions (Sec. 3-10).

Although a general solution is unattainable, particular exact solutions of Navier–Stokes are still being found. Here are some recent examples: a porous channel with moving walls, Dauenhauer and Majdalani (2003); oscillating flow in a rectangular duct, Tsangaris and Vlachakis (2003); chaotic flow in cylindrical tubes, Blyth et al. (2003); generalized Beltrami flows, Wang (1990); an unsteady stretching surface, Smith (1994); free shear layers, Varley and Seymour (1994); and two interacting vortices, Agullo and Verga (1997). For further study of laminar viscous flows, see the monographs by Constantinescu (1995), Ockendon and Ockendon (1995), and Papanastasiou et al. (1999).

3-2 COUETTE FLOWS DUE TO MOVING SURFACES

These flows are named in honor of M. Couette (1890), who performed experiments on the flow between a fixed and moving concentric cylinder. We consider several examples.

3-2.1 Steady Flow between a Fixed and a Moving Plate

In Fig. 3-1, two infinite plates are $2h$ apart, and the upper plate moves at speed U relative to the lower. The pressure $\hat{p}$ is assumed constant. The upper plate is held at temperature T_1 and the lower plate at T_0. These boundary conditions are independent of x or z ("infinite plates"); hence it follows that $u = u(y)$ and $T = T(y)$. Equations (3-1) to (3-3) reduce to

Continuity: $$\frac{\partial u}{\partial x} = 0$$

Momentum: $$0 = \mu \frac{d^2 u}{dy^2} \qquad (3\text{-}4)$$

Energy: $$0 = k\frac{d^2 T}{dy^2} + \mu \left(\frac{du}{dy}\right)^2 \qquad (3\text{-}5)$$

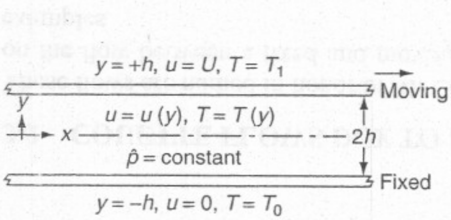

FIGURE 3-1
Couette flow between parallel plates.