Differential Expressions in Cartesian Coordinates

Velocity: $\vec{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$

Gradient: $\nabla p = \frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}$

Laplacian: $\nabla^2 \vec{V} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(u\mathbf{i} + v\mathbf{j} + w\mathbf{k})$

Divergence: $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

Curl: $\nabla \times \vec{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k}$

Continuity: $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Incompressible Navier-Stokes Eq.: $\frac{\mathrm{D}\vec{V}}{\mathrm{D}t} \equiv \frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \nabla\right)\vec{V} = -\frac{\nabla p}{\rho} + \vec{g} + \nu \nabla^2 \vec{V}$

***x*-momentum:** $\rho \frac{\mathrm{D}u}{\mathrm{D}t} = -\frac{\partial p}{\partial x} + \mu \nabla^2 u + \rho g_x$

***y*-momentum:** $\rho \frac{\mathrm{D}v}{\mathrm{D}t} = -\frac{\partial p}{\partial y} + \mu \nabla^2 v + \rho g_y$

***z*-momentum:** $\rho \frac{\mathrm{D}w}{\mathrm{D}t} = -\frac{\partial p}{\partial z} + \mu \nabla^2 w + \rho g_z$

Substantial Derivative: $\frac{\mathrm{D}A}{\mathrm{D}t} = \frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + w\frac{\partial A}{\partial z}$

Laplacian: $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Viscous Shear Stresses: $\tau_{xx} = 2\mu \frac{\partial u}{\partial x}$ $\tau_{yy} = 2\mu \frac{\partial v}{\partial y}$ $\tau_{zz} = 2\mu \frac{\partial w}{\partial z}$

Off-Diagonal: $\tau_{xy} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$ $\tau_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$ $\tau_{yz} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$

Two-Dimensional Cartesian Coordinates

Continuity: $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

***x*-momentum:**

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \rho g_x$$

***y*-momentum:**

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \rho g_y$$

Viscous Shear Stresses: $\tau_{xx} = 2\mu \frac{\partial u}{\partial x}$ $\tau_{yy} = 2\mu \frac{\partial v}{\partial y}$ $\tau_{zz} = 0$

Off-Diagonal: $\tau_{xy} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$ $\tau_{xz} = 0$ $\tau_{yz} = 0$