

Expressions in Steady Two-Dimensional Cartesian Coordinates

Steady Flow: $\frac{\partial A}{\partial t} = 0, \quad \forall A$

2D Motion in xy Plane: $w = 0$

2D Motion in xy Plane: $\frac{\partial A}{\partial z} = 0, \quad \forall A$

Continuity: $\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Navier-Stokes x-momentum: $\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x$

Navier-Stokes y-momentum: $\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y$

Navier-Stokes z-momentum: $0 = 0$

Velocity: $\mathbf{V} = u\mathbf{i} + v\mathbf{j}$

Gravity: $\mathbf{g} = g_x\mathbf{i} + g_y\mathbf{j}$

Differential Operator: $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j}$

Gradient of a Scalar: $\nabla p = \frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j}$

Laplacian of a Vector: $\nabla^2 \mathbf{V} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (u\mathbf{i} + v\mathbf{j})$

Divergence of a Vector: $\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

Curl of a Vector: $\nabla \times \mathbf{V} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \mathbf{k}$

Stream Function Definition: $u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}$

Stream Function Equation: $\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}$ from $\nabla \times \mathbf{V} = \mathbf{0}$

Potential Function Definition: $u = -\frac{\partial \phi}{\partial x} \quad v = -\frac{\partial \phi}{\partial y}$ from $\mathbf{V} = -\nabla \phi$

Potential Function Equation: $\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}$ from $\nabla \cdot \mathbf{V} = 0$

Viscous Shear Stresses: $\tau_{xx} = 2\mu \frac{\partial u}{\partial x} \quad \tau_{yy} = 2\mu \frac{\partial v}{\partial y} \quad \tau_{zz} = 0$

Off-Diagonal: $\tau_{xy} = \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \quad \tau_{xz} = 0 \quad \tau_{yz} = 0$