Expressions in Cartesian Coordinates

Velocity: $\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$ **Gravity:** $\mathbf{g} = g_x\mathbf{i} + g_y\mathbf{j} + g_z\mathbf{k}$

Differential Operator: $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$ **Gradient:** $\nabla p = \frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}$

Laplacian of a Vector: $\nabla^2\mathbf{V} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(u\mathbf{i} + v\mathbf{j} + w\mathbf{k})$

Divergence: $\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

Curl: $\nabla \times \mathbf{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k}$

Continuity (Incompressible Mass Conservation): $\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Incompressible Navier-Stokes Equation: $\frac{\mathrm{D}\mathbf{V}}{\mathrm{D}t} \equiv \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V} = -\frac{\nabla p}{\rho} + \mathbf{g} + \nu\nabla^2\mathbf{V}$

Using Kinematic Viscosity: $\nu = \frac{\mu}{\rho}$

Navier-Stokes *x*-momentum: $\rho\frac{\mathrm{D}u}{\mathrm{D}t} = -\frac{\partial p}{\partial x} + \mu\nabla^2 u + \rho g_x$

Navier-Stokes *y*-momentum: $\rho\frac{\mathrm{D}v}{\mathrm{D}t} = -\frac{\partial p}{\partial y} + \mu\nabla^2 v + \rho g_y$

Navier-Stokes *z*-momentum: $\rho\frac{\mathrm{D}w}{\mathrm{D}t} = -\frac{\partial p}{\partial z} + \mu\nabla^2 w + \rho g_z$

Substantial Derivative: $\frac{\mathrm{D}A}{\mathrm{D}t} = \frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + w\frac{\partial A}{\partial z}$

Laplacian: $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Stream Function Definition: $u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}$ (in 2-D only)

Stream Function Equation: $\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$ from $\nabla \times \mathbf{V} = \mathbf{0}$

Potential Function Definition: $u = -\frac{\partial \phi}{\partial x} \quad v = -\frac{\partial \phi}{\partial y} \quad w = -\frac{\partial \phi}{\partial z}$ from $\mathbf{V} = -\nabla\phi$

Potential Function Equation: $\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$ from $\nabla \cdot \mathbf{V} = 0$

Viscous Shear Stresses: $\tau_{xx} = 2\mu\frac{\partial u}{\partial x} \quad \tau_{yy} = 2\mu\frac{\partial v}{\partial y} \quad \tau_{zz} = 2\mu\frac{\partial w}{\partial z}$

Off-Diagonal: $\tau_{xy} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \quad \tau_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \quad \tau_{yz} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$