Expressions in Steady Two-Dimensional Cylindrical Coordinates

Steady Flow: $\dfrac{\partial A}{\partial t} = 0, \quad \forall A$

2D Motion in *r*-θ Plane: $V_z = 0$

2D Motion in *r*-θ Plane: $\dfrac{\partial A}{\partial z} = 0, \quad \forall A$

Continuity: $\nabla \cdot \mathbf{V} = \dfrac{1}{r}\dfrac{\partial}{\partial r}(rV_r) + \dfrac{1}{r}\dfrac{\partial V_\theta}{\partial \theta} \equiv \dfrac{\partial V_r}{\partial r} + \dfrac{V_r}{r} + \dfrac{1}{r}\dfrac{\partial V_\theta}{\partial \theta} = 0$

NS r-momentum:

$$\rho\left(V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} - \frac{V_r}{r^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right) + \rho g_r$$

NS θ-mom.: $\rho\left(V_r\dfrac{\partial V_\theta}{\partial r} + \dfrac{V_\theta}{r}\dfrac{\partial V_\theta}{\partial \theta} + \dfrac{V_r V_\theta}{r}\right) = -\dfrac{1}{r}\dfrac{\partial p}{\partial \theta} + \mu\left(\dfrac{\partial^2 V_\theta}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial V_\theta}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 V_\theta}{\partial \theta^2} - \dfrac{V_\theta}{r^2} + \dfrac{2}{r^2}\dfrac{\partial V_r}{\partial \theta}\right) + \rho g_\theta$

NS *z*-momentum: $0 = 0$

Velocity: $\mathbf{V} = V_r\mathbf{e}_r + V_\theta\mathbf{e}_\theta$

Gravity: $\mathbf{g} = g_r\mathbf{e}_r + g_\theta\mathbf{e}_\theta$

Differential Operator: $\nabla = \dfrac{\partial}{\partial r}\mathbf{e}_r + \dfrac{1}{r}\dfrac{\partial}{\partial \theta}\mathbf{e}_\theta$

Gradient of a Scalar: $\nabla p = \dfrac{\partial p}{\partial r}\mathbf{e}_r + \dfrac{1}{r}\dfrac{\partial p}{\partial \theta}\mathbf{e}_\theta$

Laplacian of a Vector: $\nabla^2\mathbf{V} = \left(\dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial \theta^2}\right)(V_r\mathbf{e}_r + V_\theta\mathbf{e}_\theta)$

Divergence of a Vector: $\nabla \cdot \mathbf{V} = \dfrac{1}{r}\dfrac{\partial}{\partial r}(rV_r) + \dfrac{1}{r}\dfrac{\partial V_\theta}{\partial \theta} \equiv \dfrac{\partial V_r}{\partial r} + \dfrac{V_r}{r} + \dfrac{1}{r}\dfrac{\partial V_\theta}{\partial \theta}$

Curl of a Vector: $\nabla \times \mathbf{V} = -\dfrac{\partial V_\theta}{\partial z}\mathbf{e}_r + \dfrac{1}{r}\left[\dfrac{\partial}{\partial r}(rV_\theta) - \dfrac{\partial V_r}{\partial \theta}\right]\mathbf{k}$

Stream Function Definition: $V_r = \dfrac{1}{r}\dfrac{\partial \psi}{\partial \theta} \quad V_\theta = -\dfrac{\partial \psi}{\partial r}$

Stream Function Equation: $\nabla^2\psi = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial \psi}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 \psi}{\partial \theta^2} = 0$ from $\nabla \times \mathbf{V} = \mathbf{0}$

Potential Function Definition: $V_r = -\dfrac{\partial \phi}{\partial r} \quad V_\theta = -\dfrac{1}{r}\dfrac{\partial \phi}{\partial \theta}$ from $\mathbf{V} = -\nabla\phi$

Potential Function Equation: $\nabla^2\phi = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial \phi}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 \phi}{\partial \theta^2} = 0$ from $\nabla \cdot \mathbf{V} = 0$

Viscous Shear Stresses: $\tau_{rr} = 2\mu\dfrac{\partial V_r}{\partial r} \quad \tau_{\theta\theta} = 2\mu\left(\dfrac{1}{r}\dfrac{\partial V_\theta}{\partial \theta} + \dfrac{V_r}{r}\right) \quad \tau_{zz} = 0$

Off-Diagonal: $\tau_{r\theta} = \mu\left[\dfrac{1}{r}\dfrac{\partial V_r}{\partial \theta} + r\dfrac{\partial}{\partial r}\left(\dfrac{V_\theta}{r}\right)\right] \quad \tau_{\theta z} = \mu\dfrac{\partial V_\theta}{\partial z} \quad \tau_{zr} = 0$