Classical Pipe Flow Problem Solving Strategies

Fundamental Energy Equation:

(Eq. 1) $$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_w+h_L$$

Energy equation applied for a steady incompressible flow between two points separated by a distance L.

$h_w=\dfrac{\dot{W}}{\dot{m}g}$; where $\dot{W}$ represents the rate at which work is removed from the fluid. It is positive and equal to the net input power to a turbine, and it is negative and equal to the net output power from a pump.

Head-Loss Equation:

(Eq. 2) $$h_L=h_M+h_m=\left(f\frac{L}{D}+f\sum\frac{L_e}{D}+\sum K\right)\frac{V^2}{2g}=\left(f\frac{L}{D}+f\sum\frac{L_e}{D}+\sum K\right)\frac{8Q^2}{\pi^2gD^4}$$

(Eq. 3) $$f=\begin{cases}\dfrac{64}{\mathrm{Re}};\ \mathrm{Re}\le 2300;\ \text{else}\\ 1.6364/\ln^2\left[\dfrac{6.9}{\mathrm{Re}}+\dfrac{(e/D)^{1.11}}{3.7}\right]\end{cases}$$

Flow-Rate Problem: When Q and V are unknown.

(1.) To start, set Re = 10^8 and find f from Eq. 3 (fully rough zone)

(2.) Find Q that satisfies Eq. 2: $Q=\dfrac{\pi D^2}{4}V=\sqrt{\dfrac{\pi^2gD^4h_L}{8\left(f\dfrac{L}{D}+f\sum\dfrac{L_e}{D}+\sum K\right)}}$

(3.) Find $$\mathrm{Re}=\frac{\rho VD}{\mu}=\frac{4\rho Q}{\pi\mu D}=\frac{4\dot{m}}{\pi\mu D}$$

(4.) Find f from Eq. 3

(5.) Repeat steps (2.)-(4.) until satisfied.

Pipe-Sizing Problem: When D is unknown.

(1.) To start, guess f = 0.03 and a value for D.

(2.) Find an update for D that satisfies Eq. 2: $D=\left[\dfrac{8Q^2\left(f\dfrac{L}{D}+f\sum\dfrac{L_e}{D}+\sum K\right)}{\pi^2gh_L}\right]^{0.25}$

(3.) Find $$\mathrm{Re}=\frac{\rho VD}{\mu}=\frac{4\rho Q}{\pi\mu D}=\frac{4\dot{m}}{\pi\mu D}$$ and (*e/D*)

(4.) Find f from Eq. 3

(5.) Repeat steps (2.)-(4.) until satisfied.