

Advanced Pipe Flow Problem Solving Strategies

Fundamental Energy Equation:

$$(Eq. 1) \quad \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_w + h_L$$

Energy equation applied to a steady incompressible flow between two points separated by a distance L .

$h_w = \frac{\dot{W}}{\dot{m}g}$; where \dot{W} represents the rate at which work is removed from the fluid. It is positive and equal to the net input power to a turbine, and it is negative and equal to the net output power from a pump.

Head-Loss Equation:

$$(Eq. 2) \quad h_L = h_M + h_m = \left(f \frac{L}{D} + f \sum \frac{L_e}{D} + \sum K \right) \frac{V^2}{2g} = \left(f \frac{L}{D} + f \sum \frac{L_e}{D} + \sum K \right) \frac{8Q^2}{\pi^2 g D^4}$$

$$(Eq. 3) \quad f = \begin{cases} \frac{64}{\text{Re}}; & \text{Re} \leq 2300; \text{ else} \\ 1.6364 / \ln^2 \left[\frac{6.9}{\text{Re}} + \frac{(e/D)^{1.11}}{3.7} \right] \end{cases}$$

Flow-Rate Problem: When Q and V are unknown.

$$(1.) \text{ Guess} \quad Q = \frac{\pi D^2}{4} V = -0.965 \sqrt{\frac{g D^5 h_L}{L}} \ln \left[\frac{e/D}{3.7} + \sqrt{\frac{3.17 \mu^2 L}{g \rho^2 D^3 h_L}} \right] \text{ (exact if } h_m = 0 \text{)}$$

$$(2.) \text{ Find} \quad \text{Re} = \frac{\rho V D}{\mu} = \frac{4 \rho Q}{\pi \mu D} = \frac{4 \dot{m}}{\pi \mu D}$$

(3.) Find f from Eq. 3

$$(4.) \text{ Find } Q \text{ that satisfies Eq. 2: } Q = \frac{\pi D^2}{4} V = \sqrt{\frac{\pi^2 g D^4 h_L}{8 \left(f \frac{L}{D} + f \sum \frac{L_e}{D} + \sum K \right)}}$$

(5.) Repeat steps (2.)-(4.) until satisfied.

Pipe-Sizing Problem: When D is unknown.

$$(1.) \text{ Guess} \quad D = 0.66 \left[e^{1.25 \left(\frac{L Q^2}{g h_L} \right)^{4.75}} + \frac{\mu}{\rho} Q^{9.4} \left(\frac{L}{g h_L} \right)^{5.2} \right]^{0.04} \text{ (exact if } h_m = 0 \text{)}$$

$$(2.) \text{ Find} \quad \text{Re} = \frac{\rho V D}{\mu} = \frac{4 \rho Q}{\pi \mu D} = \frac{4 \dot{m}}{\pi \mu D} \text{ and } (e/D)$$

(3.) Find f from Eq. 3

$$(4.) \text{ Find an update for } D \text{ that satisfies Eq. 2: } D = \left[\frac{8 Q^2 \left(f \frac{L}{D} + f \sum \frac{L_e}{D} + \sum K \right)}{\pi^2 g h_L} \right]^{0.25}$$

(5.) Repeat steps (2.)-(4.) until satisfied.