

Dimensional Analysis

FORCE $F : \text{MLt}^{-2}$	$\mu : \text{ML}^{-1}\text{t}^{-1}$	$\rho : \text{ML}^{-3}$	DIAMETER $D : \text{L}$	SPEED $V : \text{Lt}^{-1}$
PRESSURE $p : \text{ML}^{-1}\text{t}^{-2}$	$\nu : \text{L}^2\text{t}^{-1}$	$g : \text{Lt}^{-2}$	f or $\omega : \text{t}^{-1}$	SURF TENSION $\sigma : \text{Mt}^{-2}$
TORQUE $T : \text{ML}^2\text{t}^{-2}$	WORK $W : \text{ML}^2\text{t}^{-2}$	AREA $A : \text{L}^2$	FLOWRATE $Q : \text{L}^3\text{t}^{-1}$	VOLUME $\forall : \text{L}^3$
MASS FLOWRATE: Mt^{-1}	POWER: ML^2t^{-3}	$\gamma = \rho g : \text{ML}^{-2}\text{t}^{-2}$	YOUNG'S MODULUS :	$E = Y : \text{ML}^{-1}\text{t}^{-2}$

Procedure to Apply the Buckingham Pi Theorem:

1. List the “ n ” parameters involved, starting with the dependent parameter.
2. Under each parameter, write the primary dimensions MLtT.
3. Find the rank “ r ” of the dimensional matrix. Typically $r =$ number of primary dimensions.
4. Select r *repeating parameters* from the n available. Avoid the *dependent variable* along with μ , c , Δp or σ . The repeating parameters must have independent units that yet include in total all the primary dimensions. The best choice is that of parameters similar to ρ , V , D .
5. For each of the $(n - r)$ remaining parameters (called *nonrepeating*), form a nondimensional Pi parameter Π starting with the dependent variable.
6. Express the Pi parameter containing the dependent variable as a function of the remaining Pi parameters: $\Pi_1 = F(\Pi_2, \Pi_3, \dots)$. Identify well-known Pi parameters, especially those that are named after famous scientists.

$$\frac{\rho V L}{\mu} = \text{Re Reynolds no.} \propto \frac{\text{Inertial Force}}{\text{Viscous Force}} \text{ near solid boundaries.}$$

$$\frac{V}{c} = \sqrt{\frac{\rho V^2}{p}} = \frac{V}{\sqrt{kRT}} = M \text{ Mach no.} \propto \frac{\text{Inertial Force}}{\text{Compressibility Force}} \text{ in high speed gaseous flows.}$$

$$\frac{\omega L}{V} \text{ or } \frac{fL}{V} = St \text{ Strouhal no.} \propto \frac{\text{Time-Dependent Inertial Force}}{\text{Time-Independent Inertial Force}} \text{ unsteady/oscillatory flows.}$$

$$L\sqrt{\frac{\omega}{\nu}} = \lambda \text{ Stokes/Womersley no.} \propto \frac{\text{Time-Dependent Inertial Force}}{\text{Viscous Force}} \text{ unsteady flows near walls.}$$

$$\frac{V}{\sqrt{gL}} \text{ or } \frac{V^2}{gL} = Fr \text{ Froude no.} \propto \frac{\text{Inertial Force}}{\text{Gravitational Force}} \text{ in free-surface flows.}$$

$$\frac{\rho V^2 L}{\sigma} = We \text{ Weber no.} \propto \frac{\text{Inertial Force}}{\text{Surface Tension Force}} \text{ in capillary flows, droplets, ripple waves.}$$

$$\frac{\mu V}{\sigma} = Ca \text{ Capillarity no.} \propto \frac{\text{Viscous Force}}{\text{Surface Tension Force}} \text{ in capillary flows near solid boundaries.}$$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = Eu \text{ Euler no.} \propto \frac{\text{Pressure Force}}{\text{Inertial Force}} \text{ important when pressure gradients drive the flow.}$$

$$\frac{p - p_v}{\frac{1}{2}\rho V^2} = C \text{ Cavitation no.} \propto \frac{\text{Pressure Force}}{\text{Inertial Force}} \text{ when pressure drops below vapor pressure.}$$

$$\frac{F_D}{\frac{1}{2}\rho V^2 A} = C_D \text{ Drag coeff.} \propto \frac{\text{Drag Force}}{\text{Inertial Force}} \text{ when drag is important.}$$

$$\frac{\rho V^3}{\mu \omega^2 L} = S_p \text{ Penetration no.} \propto \frac{\text{Time-Dependent Inertial Force}}{\text{Viscous Force}} \text{ oscillating flow over porous}$$

walls.