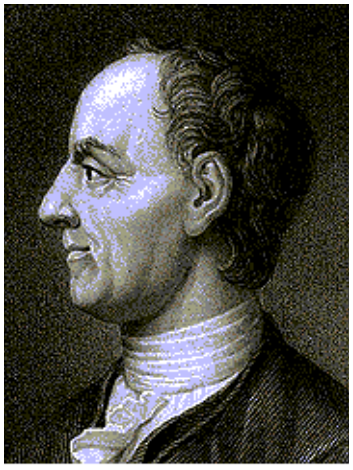

The most significant mathematician of all time, Leonhard Euler was born in Basel in 1707. He contributed to areas of both pure and applied mathematics, including calculus, analysis, number theory, topology, algebra, geometry, trigonometry, analytical mechanics, hydrodynamics, and the theory of the moon's motion.

Euler's father was a clergyman who, like Jacques Bernoulli's father, hoped that his son would enter the ministry. However, the young man studied under Jean Bernoulli and associated with his sons, Nicolaus and Daniel, and through them discovered his vocation. The elder Euler also was adept in mathematics, having been a pupil under Jacques Bernoulli, and helped to instruct the son in the elements of the subject, despite his hope that Leonhard would pursue a theological career. At all events, the young man was broadly trained, for to the study of mathematics he added theology, medicine, astronomy, physics, and oriental languages. This breadth stood him in good stead when in 1727 he heard from Russia that there was an opening in medicine in the St. Petersburg Academy, where the young Bernoullis had gone as professors of mathematics. This important institution had been established only a few years earlier by Catherine I along lines laid down by her late husband, Peter the Great, with the advice of Leibniz. On the recommendation of the Bernoullis, two of the brightest luminaries in the earliest days of the Academy, Euler was called to be a member of the section on medicine and physiology; but on the very day that he arrived in Russia, Catherine died. The fledgling Academy nearly succumbed with her, because the new rulers showed less sympathy for learned foreigners than had Peter and Catherine.

The Academy somehow managed to survive, and Euler, in 1730, found himself in the chair of natural philosophy rather than in the medical section. His friend Nicolaus Bernoulli had died, by drowning, in St. Petersburg the year before Euler arrived, and in 1733, Daniel Bernoulli left Russia to occupy the chair in mathematics at Basel. Thereupon Euler, at the age of twenty-six, became the Academy's chief mathematician.

He married and settled down to pursue in earnest mathematical research and rear a family that ultimately included thirteen children. The St. Petersburg Academy had established a research journal, the *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, and almost from the start, Euler contributed a spate of mathematical articles. The editors did not have to worry about a shortage of material as long as the pen of Euler was busy.

It was said by the French academician François Arago that Euler could calculate without any apparent effort, "just as men breathe, as eagles sustain themselves in the air." As a result, Euler composed mathematical memoirs while playing with his children. In 1735 he had lost the sight of his right eye -through overwork, it is said- but this misfortune in no way diminished the rate of output of his research. He is supposed to have said his pencil seemed to surpass him in intelligence, so easily did memoirs flow.

He published more than 500 books and papers during his lifetime. For almost half a century after his death, works by Euler continued to appear in the publications of the St. Petersburg Academy and all around the world. A bibliographical list of Euler's works, including posthumous items, contains 886 entries; and it is estimated that his collected works, now being published, will run close to seventy-five substantial volumes. His mathematical research during his lifetime averaged about 800 pages a year; no mathematician has ever exceeded the output of this man whom Arago characterized as "Analysis Incarnate."

Euler early acquired an international reputation; even before leaving Basel, he had received an honorable mention from the Parisian *Académie des Sciences* for an essay on the masting of ships. In later years he frequently entered essays in the contests set by the *Académie*, and twelve times he won the coveted biennial prize. The topics ranged widely, and on one occasion, in 1724, Euler shared with Maclaurin and Daniel Bernoulli a prize for an essay on the tides.

Euler was never guilty of false pride, and he wrote works on all levels, including textbook material for use in the Russian schools. He generally wrote in Latin, and sometimes in French, although German was his native tongue. Euler had unusual language facility and never encountered a language problem.

In 1741, Euler was invited by Frederick the Great to join the Berlin Academy, and the invitation was accepted. Euler spent twenty-five years at Frederick's court. Euler's stay at Berlin was not entirely happy, for Frederick preferred a scholar who scintillated, as did Voltaire. The monarch, who valued philosophers above geometers, referred to the unsophisticated Euler as a "mathematical cyclops," and relationships at the court became intolerable for Euler. Catherine the Great was only too eager to have the prolific mathematician resume his place in the St. Petersburg Academy, and in 1766 Euler returned to Russia. During this year, Euler learned that he was losing by cataract the sight of his remaining eye, and he prepared for ultimate blindness by practicing writing with chalk on a large slate and by dictating to his children. An operation was performed in 1771, and for a few days, Euler saw once more; but success was short-lived and Euler spent almost all of the last seventeen years of his life in total darkness. Even this tragedy failed to stem the flood of his research and publication, which continued unabated until in 1783, at the age of seventy-six, he suddenly died while sipping tea and enjoying the company of one of his grandchildren.

From 1727 to 1783 the pen of Euler had been busy adding to knowledge in virtually every branch of pure and applied mathematics, from the most elementary to the most advanced. Moreover, in most respects, Euler wrote the language and notations we use today, for no other individual was so largely responsible for the form of college-level mathematics today as was Euler, the most successful notation builder of all times. A minute fraction of the symbols attributed to Euler are $\pi$, $e^x$, $\log$, $\ln$, $i=\sqrt{-1}$, $\sum$, $\sin x$, $\cos x$, $\tan x$, $\sec x$, etc.

# *More on Euler*

Euler is considered to be one of the founders of pure mathematics. He not only made decisive and formative contributions to the subjects of geometry, calculus, mechanics, and number theory but also developed methods for solving problems in observational astronomy and demonstrated useful applications of mathematics in technology and public affairs.

Euler's mathematical ability earned him the esteem of Johann or Jean Bernoulli, one of the first mathematicians in Europe at that time, and of his sons Daniel and Nicolas. In 1727 he moved to St. Petersburg, where he became an associate of the St. Petersburg Academy of Sciences and in 1733 succeeded Daniel Bernoulli to the chair of mathematics.

By means of his numerous books and memoirs that he submitted to the academy, Euler carried integral calculus to a higher degree of perfection, developed the theory of trigonometric and logarithmic functions, reduced analytical operations to a greater simplicity, and threw new light on nearly all parts of pure mathematics. Overtaxing himself, Euler in 1735 lost the sight of one eye. Then, invited by Frederick the Great in 1741, he became a member of the Berlin Academy, where for 25 years he produced a steady stream of publications, many of which he contributed to the St. Petersburg Academy, which granted him a pension. In 1748, in his Introductio in analysin infinitorum, he developed the concept of function in mathematical analysis, through which variables are related to each other and in which he advanced the use of infinitesimals and infinite quantities. He did for modern analytic geometry and trigonometry what the Elements of Euclid had done for ancient geometry, and the resulting tendency to render mathematics and physics in arithmetical terms has continued ever since. He is known for familiar results in elementary geometry; for example, the Euler line through the orthocentre (the intersection of the altitudes in a triangle), the circumcentre (the centre of the circumscribed circle of a triangle), and the barycentre (the "centre of gravity," or centroid) of a triangle. He was responsible for treating trigonometric functions--i.e., the relationship of an angle to two sides of a triangle--as numerical ratios rather than as lengths of geometric lines and for relating them, through the so-called Euler identity $(e^{ix} = \cos x + i \sin x)$, with complex numbers (e.g., $1 + \sqrt{-1}$). He discovered the imaginary logarithms of negative numbers and showed that each complex number has an infinite number of logarithms.

Euler's textbooks in calculus, Institutiones calculi differentialis in 1755 and Institutiones calculi integralis in 1768-70, have served as prototypes to the present because they contain formulas of differentiation and numerous methods of indefinite integration, many of which he invented himself, for determining the work done by a force and for solving geometric problems; and he made advances in the theory of linear differential equations, which are useful in solving problems in physics. Thus, he enriched mathematics with substantial new concepts and techniques. He introduced many current notations, such as $\sum$ for the sum; the symbol $e$ for the base of natural logarithms; a, b, and c for the sides of a triangle and A, B, and C for the opposite angles; the letter $f$ and parentheses for a function; the use of the symbol $\pi$ for the ratio of circumference to diameter in a circle; and $i$ for $\sqrt{-1}$.

After Frederick the Great became less cordial toward him, Euler in 1766 accepted the invitation of Catherine II to return to Russia. Soon after his arrival at St. Petersburg, a cataract formed in his remaining good eye, and he spent the last years of his life in total blindness. Despite this tragedy, his productivity continued undiminished, sustained by an uncommon memory and a remarkable facility in mental computations. His interests were broad, and his Lettres à une princesse

d'Allemagne in 1768-72 were an admirably clear exposition of the basic principles of mechanics, optics, acoustics, and physical astronomy. Not a classroom teacher, Euler nevertheless had a more pervasive pedagogical influence than any modern mathematician. He had few disciples, but he helped to establish mathematical education in Russia.

Euler devoted considerable attention to developing a more perfect theory of lunar motion, which was particularly troublesome, since it involved the so-called three-body problem--the interactions of Sun, Moon, and Earth. (The problem is still unsolved.) His partial solution, published in 1753, assisted the British Admiralty in calculating lunar tables, of importance then in attempting to determine longitude at sea. One of the feats of his blind years was to perform all the elaborate calculations in his head for his second theory of lunar motion in 1772. Throughout his life Euler was much absorbed by problems dealing with the theory of numbers, which treats of the properties and relationships of integers, or whole numbers (0, +/-1, +/-2, etc.); in this, his greatest discovery, in 1783, was the law of quadratic reciprocity, which has become an essential part of modern number theory.

In his effort to replace synthetic methods by analytic ones, Euler was succeeded by J.-L. Lagrange. But, where Euler had delighted in special concrete cases, Lagrange sought for abstract generality; and, while Euler incautiously manipulated divergent series, Lagrange attempted to establish infinite processes upon a sound basis. Thus it is that Euler and Lagrange together are regarded as the greatest mathematicians of the 18th century; but Euler has never been excelled either in productivity or in the skillful and imaginative use of algorithmic devices (i.e., computational procedures) for solving problems.