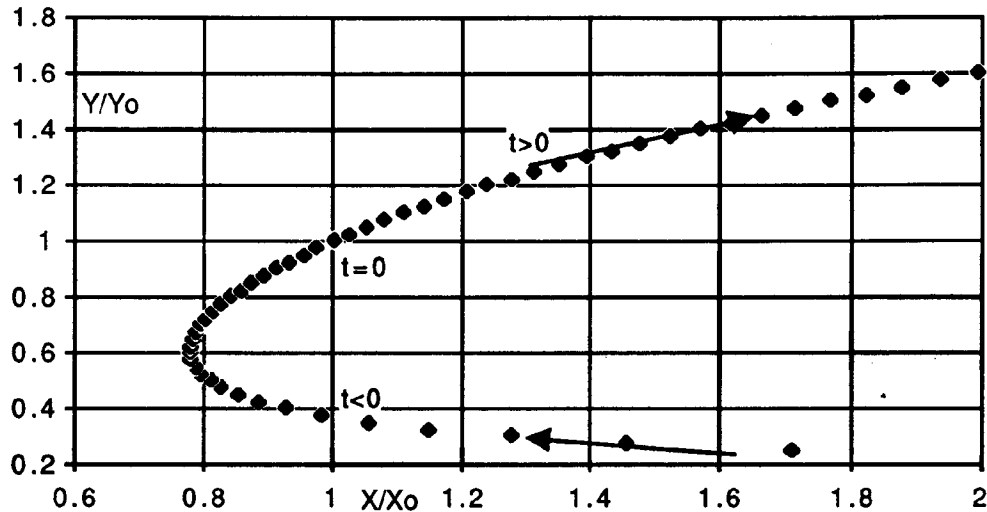


as *planned* for this problem, that  $u$  did not depend upon  $y$  and  $v$  did not depend upon  $x$ .] Now eliminate  $t$  between these two to get a geometric expression for this particular pathline:

$$x = x_0 \exp\{\ln(y/y_0) + \ln^2(y/y_0)\}$$

This pathline is shown in the sketch below.



**1.85-a** Report to the class on the achievements of *Evangelista Torricelli*.

**SOLUTION:** Torricelli's biography is taken from a goldmine of information which I did not put in the references, preferring to let the students find it themselves: C. C. Gillespie (ed.), *Dictionary of Scientific Biography*, 15 vols., Charles Scribner's Sons, New York, 1976.

Torricelli (1608-1647) was born in Faenza, Italy to poor parents who recognized his genius and arranged through Jesuit priests to have him study mathematics, philosophy, and (later) hydraulic engineering under Benedetto Castelli. His work on dynamics of projectiles attracted the attention of Galileo himself, who took on Torricelli as an assistant in 1641. Galileo died one year later, and Torricelli was appointed in his place as "mathematician and philosopher" by Duke Ferdinando II of Tuscany. He then took up residence in Florence, where he spent his five happiest years, until his death in 1647. In 1644 he published his only known printed work, *Opera Geometrica*, which made him famous as a mathematician and geometer.

In addition to many contributions to geometry and calculus, Torricelli was the first to show that a zero-drag projectile formed a *parabolic* trajectory. His tables of trajectories for various angles and initial velocities were used by Italian artillerymen. He was an excellent machinist and constructed - and sold - the very finest telescope lenses in Italy.

Torricelli's hydraulic studies were brief but stunning, leading Ernst Mach to proclaim him the 'founder of hydrodynamics'. He deduced his theorem that the velocity of efflux from a hole in a tank was equal to  $\sqrt{2gh}$ , where  $h$  is the height of the free surface above the hole. He also showed that the efflux jet was parabolic and even commented on water-droplet breakup and the effect of air resistance. By experimenting with various liquids in closed tubes - including mercury (from mines in Tuscany) - he thereby invented the *barometer*. From barometric pressure (about 30 feet of water) he was able to explain why siphons did not work if the elevation change was too large. He also was the first to explain that winds were produced by temperature and *density differences* in the atmosphere and not by "evaporation".

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**1.85-b** Report to the class on the achievements of *Henri de Pitot*.

**SOLUTION:** The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a). Pitot (1695-1771) was born in Aramon, France to patrician parents. He hated to study and entered the military instead, but only for a short time. Chance reading of a textbook obtained in Grenoble led him back to academic studies of mathematics, astronomy, and engineering. In 1723 he became assistant to Réamur at the French Academy of Sciences and in 1740 became a civil engineer upon his appointment as a director of public works in Languedoc Province. He retired in 1756 and returned to Aramon until his death in 1771.

Pitot's research was apparently mediocre, described as "competent solutions to minor problems without lasting significance" - not a good recommendation for tenure nowadays! His *lasting* contribution was the invention, in 1735, of the instrument which bears his name: a glass tube bent at right angles and inserted into a moving stream with the opening facing upstream. The water level in the tube rises a distance  $h$  above the surface, and Pitot correctly deduced that the stream velocity  $\approx \sqrt{2gh}$ . This is still a basic instrument in fluid mechanics.

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**1.85c** Report to the class on the achievements of *Antoine Chézy*.

**SOLUTION:** The following notes are from Rouse and Ince [Ref. 23]. Chézy (1718-1798) was born in Châlons-sur-Marne, France, studied engineering at the Ecole des Ponts et Chaussées and then spent his entire career working for this school, finally being appointed Director one year before his death. His chief contribution was to study the flow in open channels and rivers, resulting in a famous formula, used even today, for the average velocity:

$$V \approx \text{const} \sqrt{AS/P}$$

where  $A$  is the cross-section area,  $S$  the bottom slope, and  $P$  the wetted perimeter, i.e., the

length of the bottom and sides of the cross-section. The “constant” depends primarily on the roughness of the channel bottom and sides. [See Chap. 10 for further details.]

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**1.85-d** Report to the class on the achievements of *Gotthilf Heinrich Ludwig Hagen*.

**SOLUTION:** The following notes are from Rouse and Ince [Ref. 23]. Hagen (1884) was born in Königsberg, East Prussia and studied there, having among his teachers the famous mathematician Bessel. He became an engineer, teacher, and writer and published a handbook on hydraulic engineering in 1841. He is best known for his study in 1839 of pipe-flow resistance, for water flow at heads of 0.7 to 40 cm, diameters of 2.5 to 6 mm, and lengths of 47 to 110 cm. The measurements indicated that the pressure drop was proportional to  $Q$  at low heads and proportional (approximately) to  $Q^2$  at higher heads, where “strong movements” occurred: turbulence. He also showed that  $\Delta p$  was approximately proportional to  $D^{-4}$ .

Later, in an 1854 paper, Hagen noted that the difference between laminar and turbulent flow was clearly visible in the efflux jet, which was either “smooth or fluctuating”, and in glass tubes, where sawdust particles either “moved axially” or, at higher  $Q$ , “came into whirling motion”. Thus Hagen was a true pioneer in fluid mechanics experimentation. Unfortunately, his achievements were somewhat overshadowed by the more widely publicized 1840 tube-flow studies of J. L. M. Poiseuille, the French physician.

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**1.85-e** Report to the class on the achievements of *Julius Weisbach*.

**SOLUTION:** The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a) and also from Rouse and Ince [Ref. 23].

Weisbach (1806-1871) was born near Annaberg, Germany, the 8th of nine children of working-class parents. He studied mathematics, physics, and mechanics at Göttingen and Vienna and in 1831 became instructor of mathematics at Freiberg Gymnasium. In 1835 he was promoted to full professor at the Bergakademie in Freiberg. He published 15 books and 59 papers, primarily on hydraulics. He was a skilled laboratory worker and summarized his results in *Experimental-Hydraulik* (Freiberg, 1855) and in the *Lehrbuch der Ingenieur- und Maschinen-Mechanik* (Brunswick 1845), which was still in print 60 years later. There were 13 chapters on hydraulics in this latter treatise. Weisbach modernized the subject of fluid mechanics, and his discussions and drawings of flow patterns would be welcome in any 20th century textbook - see Rouse and Ince [23] for examples.

Weisbach was the first to write the pipe-resistance head-loss formula in modern form:  $h_{f(\text{pipe})} = f(L/D)(V^2/2g)$ , where  $f$  was the dimensionless ‘friction factor’, which Weisbach noted was not a constant but related to the pipe flow parameters [see Sect. 6.4]. He was also

the first to derive the “weir equation” for volume flow rate  $Q$  over a dam of crest length  $L$ :

$$Q \approx \frac{2}{3} C_w (2g)^{1/2} \left[ \left( H + \frac{V^2}{2g} \right)^{3/2} - \left( \frac{V^2}{2g} \right)^{3/2} \right] \approx \frac{2}{3} C_w (2g)^{1/2} H^{3/2}$$

where  $H$  is the upstream water head level above the dam crest and  $C_w$  is a dimensionless weir coefficient  $\approx O(\text{unity})$ . [see Sect. 10.7] In 1860 Weisbach received the first Honorary Membership awarded by the German engineering society, the *Verein Deutscher Ingenieure*.

**1.85-f** Report to the class on the achievements of *George Gabriel Stokes*.

**SOLUTION:** The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a). Stokes (1819-1903) was born in Skreen, County Sligo, Ireland to a clerical family associated for generations with the Church of Ireland. He attended Bristol College and Cambridge University and, upon graduation in 1841, was elected Fellow of Pembroke College, Cambridge. In 1849, he became Lucasian Professor at Cambridge, a post once held by Isaac Newton. His 60-year career was spent primarily at Cambridge and resulted in many honors: President of the Cambridge Philosophical Society (1859), secretary (1854) and president (1885) of the Royal Society of London, member of Parliament (1887-1891), knighthood (1889), the Copley Medal (1893), and Master of Pembroke College (1902). A true ‘natural philosopher’, Stokes systematically explored hydrodynamics, elasticity, wave mechanics, diffraction, gravity, acoustics, heat, meteorology, and chemistry. His primary research output was from 1840-1860, for he later became tied down with administrative duties.

In hydrodynamics, Stokes has several formulas and fields named after him:

- (1) The equations of motion of a linear viscous fluid: the *Navier-Stokes equations*.
- (2) The motion of nonlinear deep-water surface waves: *Stokes waves*.
- (3) The drag on a sphere at low Reynolds number: *Stokes’ formula*,  $F = 3\pi\mu VD$ .
- (4) Flow over immersed bodies for  $Re \ll 1$ : *Stokes flow*.
- (5) A metric (CGS) unit of kinematic viscosity,  $\nu$ :  $1 \text{ cm}^2/\text{s} = 1 \text{ stoke}$ .
- (6) A relation between the 1st and 2nd coefficients of viscosity: *Stokes’ hypothesis*.
- (7) A streamfunction for axisymmetric flow: *Stokes’ stream function* [see Chap. 8].

Although Navier, Poisson, and Saint-Venant had made derivations of the equations of motion of a viscous fluid in the 1820’s and 1830’s, Stokes was quite unfamiliar with the French literature. He published a completely independent derivation in 1845 of the *Navier-Stokes equations* [see Sect. 4.3], using a ‘continuum-calculus’ rather than a ‘molecular’ viewpoint, and showed that these equations were directly analogous to the motion of elastic solids. Although not really new, Stokes’ equations were notable for being the first to replace the mysterious French ‘molecular coefficient’  $\epsilon$  by the coefficient of absolute viscosity,  $\mu$ .

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**1.85-g** Report to the class on the achievements of *Moritz Weber*.

**SOLUTION:** The following notes are from Rouse and Ince [Ref. 23]. Weber (1871-1951) was professor of naval mechanics at the Polytechnic Institute of Berlin. He clarified the principles of similitude (dimensional analysis) in the form used today. It was he who named the Froude number and the Reynolds number in honor of those workers. In a 1919 paper, he developed a dimensionless surface-tension (capillarity) parameter [see Sect. 5.4] which was later named the *Weber number* in his honor.

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**1.85h** Report to the class on the achievements of *Theodor von Kármán*.

**SOLUTION:** The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a). Another good reference is his ghost-written (by Lee Edson) autobiography, *The Wind and Beyond*, Little-Brown, Boston, 1967.

Kármán (1881-1963) was born in Budapest, Hungary to distinguished and well-educated parents. He attended the Technical University of Budapest and in 1906 received a fellowship to Göttingen, where he worked for six years with Ludwig Prandtl, who had just developed boundary layer theory. He received a doctorate in 1912 from Göttingen and was then appointed director of aeronautics at the Polytechnic Institute of Aachen. He remained at Aachen until 1929, when he was named director of the newly formed Guggenheim Aeronautical Laboratory at the California Institute of Technology. Kármán developed CalTech into a premier research center for aeronautics. His leadership spurred the growth of the aerospace industry in southern California. He helped found the Jet Propulsion Laboratory and the Aerojet General Corporation. After World War II, Kármán founded a research arm for NATO, the Advisory Group for Aeronautical Research and Development, whose renowned educational institute in Brussels is now called the Von Kármán Center.

Kármán was uniquely skilled in integrating physics, mathematics, and fluid mechanics into a variety of phenomena. His most famous paper was written in 1912 to explain the puzzling alternating vortices shed behind cylinders in an steady-flow experiment conducted by K. Hiemenz, one of Kármán's students - these are now called *Kármán vortex streets* [see Fig. 5.2a]. Shed vortices are thought to have caused the destruction by winds of the Tacoma Narrows Bridge in 1940 in Washington state.

Kármán wrote 171 articles and 5 books and his methods had a profound influence on fluid mechanics education in the 20th century.

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**1.85-i** Report to the class on the achievements of *Paul Richard Heinrich Blasius*.

**SOLUTION:** The following notes are from Rouse and Ince [Ref. 23]. Blasius (1883-1970) was Ludwig Prandtl's first graduate student at Göttingen. His 1908 dissertation gave the analytic solution for the laminar boundary layer on a flat plate [see Sect. 7.4]. Then, in two papers in 1911 and 1913, he gave the first demonstration that pipe-flow resistance could be nondimensionalized as a plot of friction factor versus Reynolds number - the first "Moody-type" chart. His correlation,  $f \approx 0.316 \text{Re}_d^{-1/4}$ , is still in use today. He later worked on analytical solutions of boundary layers with variable pressure gradients.

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**1.85-j** Report to the class on the achievements of *Ludwig Prandtl*.

**SOLUTION:** The following notes are from Rouse and Ince [Ref. 23].

Ludwig Prandtl (1875-1953) is described by Rouse and Ince [23] as the father of modern fluid mechanics. Born in Munich, the son of a professor, Prandtl studied engineering and received a doctorate in elasticity. But his first job as an engineer made him aware of the lack of correlation between theory and experiment in fluid mechanics. He conducted research from 1901-1904 at the Polytechnic Institute of Hanover and presented a seminal paper in 1904, outlining the new concept of "boundary layer theory". He was promptly hired as professor and director of applied mechanics at the University of Göttingen, where he remained throughout his career. He, and his dozens of famous students, started a new "engineering science" of fluid mechanics, emphasizing (1) mathematical analysis based upon physical reasoning; (2) new experimental techniques; and (3) new and inspired flow-visualization schemes which greatly increased our understanding of flow phenomena.

In addition to boundary-layer theory, Prandtl made important contributions to (1) wing theory; (2) turbulence modeling; (3) supersonic flow; (4) dimensional analysis; and (5) instability and transition of laminar flow. He was a legendary engineering professor.

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**1.85-k** Report to the class on the achievements of *Osborne Reynolds*.

**SOLUTION:** The following notes are from Rouse and Ince [Ref. 23].

Osborne Reynolds (1842-1912) was born in Belfast, Ireland to a clerical family and studied mathematics at Cambridge University. In 1868 he was appointed chair of engineering at a college which is now known as the University of Manchester Institute of Science and Technology (UMIST). He wrote on wide-ranging topics - mechanics, electricity, navigation - and developed a new hydraulics laboratory at UMIST. He was the first person to demonstrate cavitation, that is, formation of vapor bubbles due to high velocity and low pressure. His most famous experiment, still performed in the undergraduate laboratory at UMIST (see Fig.

6.5 in the text) demonstrated transition of laminar pipe flow into turbulence. He also showed in this experiment that the viscosity was very important and led him to the dimensionless stability parameter  $\rho VD/\mu$  now called the *Reynolds number* in his honor. Perhaps his most important paper, in 1894, extended the Navier-Stokes equations (see Eqs. 4.38 of the text) to time-averaged randomly fluctuating turbulent flow, with a result now called the *Reynolds equations* of turbulence. Reynolds also contributed to the concept of the *control volume* which forms the basis of integral analysis of flow (Chap. 3).

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**1.85-l** Report to the class on the achievements of *John William Strutt, Lord Rayleigh*.

SOLUTION: The following notes are from Rouse and Ince [Ref. 23].

John William Strutt (1842-1919) was born in Essex, England and inherited the title Lord Rayleigh. He studied at Cambridge University and was a traditional hydrodynamicist in the spirit of Euler and Stokes. He taught at Cambridge most of his life and also served as president of the Royal Society. He is most famous for his work (and his textbook) on the theory of sound. In 1904 he won the Nobel Prize for the discovery of argon gas. He made at least five important contributions to hydrodynamics: (1) the equations of bubble dynamics in liquids, now known as *Rayleigh-Plesset theory*; (2) the theory of nonlinear surface waves; (3) the capillary (surface tension) instability of jets; (4) the “heat-transfer analogy” to laminar flow; and (5) dimensional similarity, especially related to viscosity data for argon gas and later generalized into group theory which previewed Buckingham’s Pi Theorem. He ended his career as president, in 1909, of the first British committee on aeronautics.

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**1.85-m** Report to the class on the achievements of *Daniel Bernoulli*.

SOLUTION: The following notes are from Rouse and Ince [Ref. 23].

Daniel Bernoulli (1700-1782) was born in Groningen, Holland, his father, Johann, being a Dutch professor. He studied at the University of Basel, Switzerland and taught mathematics for a few years at St. Petersburg, Russia. There he wrote, and published in 1738, his famous treatise *Hydrodynamica*, for which he is best known. This text contained numerous ingenious drawings illustrating various flow phenomena. Bernoulli used energy concepts to establish proportional relations between kinetic and potential energy, with pressure work added only in the abstract. Thus he never actually derived the famous equation now bearing his name (Eq. 3.77 of the text), later derived in 1755 by his friend Leonhard Euler. Daniel Bernoulli never married and thus never contributed additional members to his famous family of mathematicians.

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**1.85-n** Report to the class on the achievements of *Leonhard Euler*.

**SOLUTION:** The following notes are from Rouse and Ince [Ref. 23].

Leonhard Euler (1707-1783) was born in Basel, Switzerland and studied mathematics under Johann Bernoulli, Daniel's father. He succeeded Daniel Bernoulli as professor of mathematics at the St. Petersburg Academy, leaving there in 1741 to join the faculty of Berlin University. He lost his sight in 1766 but continued to work, aided by a prodigious memory, and produced a vast output of scientific papers, dealing with mathematics, optics, mechanics, hydrodynamics, and celestial mechanics (for which he is most famous today). His famous paper of 1755 on fluid flow derived the full inviscid equations of fluid motion (Eqs. 4.36 of the text) now called *Euler's equations*. He used a fixed coordinate system, now called the *Eulerian frame* of reference. The paper also presented, for the first time, the correct form of Bernoulli's equation (Eq. 3.77 of the text). Separately, in 1754 he produced a seminal paper on the theory of reaction turbines, leading to *Euler's turbine equation* (Eq. 11.11 of the text).

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