PROBLEM 6-7

GIVEN: WSR with $\dot{m} = 0.5$ kg/s, $V = 0.003\ m^3$

$T_{in} = 298$, $P = 1$ atm, $\dot{Q} = -2000$ W

	Y_{in}	Y_{out}	h_f°	c_p	MW
Fuel	0.2	0.001	−2000	1.100	29
Oxidizer	0.8	0.003	0	1.100	29
Products			−4000	1.100	29
			kJ/kg	kJ/kg-K	kg/kmol

FIND: T_{out}, $\tilde{\tau}_R$

ASSUMPTIONS: Simplified thermo props. as given above; $T_{ref} = 298$.

SOLUTION: Apply $\Sigma Y_i = 1$ to find Y_{Pr}:

$Y_{Pr,in} = 1 - 0.2 - 0.8 = 0$ $\qquad$ $Y_{Pr,out} = 1 - 0.001 - 0.003 = 0.996$

Apply energy conservation to find T ($= T_{out}$) (Eqn. 6.35):

$$\dot{Q}/\dot{m} = Y_{Pr,o} h_{Pr}(T) + Y_{F,o} h_F(T) + Y_{Ox,o} h_{Ox}(T) - Y_{F,in} h_F(298) - Y_{Ox,in} h_{Ox}(298)$$

Substitute caloric eqn. of state:

$$\dot{Q}/\dot{m} = Y_{Pr,o}(h^\circ_{f,Pr} + c_p(T-298)) + Y_{F,o}(h^\circ_{f,F} + c_p(T-298)) + Y_{Ox,o}(\cancelto{0}{h^\circ_{f,Ox}} + c_p(T-298)) - Y_{F,in} h^\circ_{f,F} - Y_{Ox,in} \cancelto{0}{h^\circ_{f,Ox}}$$

$$= Y_{Pr,o} h^\circ_{f,Pr} + (Y_{F,o} - Y_{F,in}) h^\circ_{f,F} + c_p(T-298)$$

Solving for T:

$$T = \frac{1}{c_p}\left[\dot{Q}/\dot{m} - Y_{Pr,o} h^\circ_{f,Pr} - (Y_{F,o} - Y_{F,in}) h^\circ_{f,F}\right] + 298$$

PROBLEM 6-7 (continued)

Substituting numerical values

$$T = \frac{1}{1.10}\left[\frac{-2.0}{0.5} - 0.996(-4000) - (0.001 - 0.2)(-2000)\right] + 298$$

$$= \frac{1}{1.10}\left(-4 + 3984 - 398\right) + 298$$

$$= 3582/1.10 + 298 \quad \boxed{= 3554\ K}$$

To find τ_R, apply Eqns. 6.37 & 6.38:

$$\tau_R = \rho V/\dot{m} = \frac{P\, MW}{R_u T} V/\dot{m}$$

$$= \frac{101{,}325(29)}{8315\,(3554)} \frac{0.003}{0.5}$$

$$\tau_R = 0.0994\,(0.006) = 0.000597$$

$$\boxed{\tau_R = 0.597\ ms}$$

COMMENT: Being given the outlet stream composition made this problem simple.

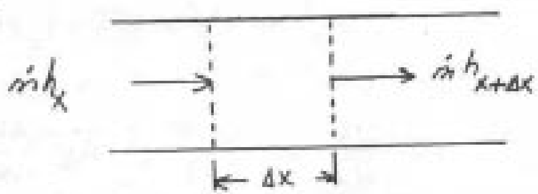
PROBLEM 6-8

GIVEN: PFR w/ $A_{x\text{-sec}}$ constant
Single-step Arrhenius reactⁿ, 3 species
Equal & constant c_p's & MWs

FIND: Conservation-of-energy expressed using T & Y_i's.

ASSUMPTIONS: Steady-state + givens

SOLUTION:



Write 1st law:

$$\underbrace{\dot{Q}}_{0} - \underbrace{\dot{W}}_{0} = \dot{m}(h_{x+\Delta x} - h_x) \quad \text{so} \quad \dot{m}\frac{dh}{dx} = 0$$

$$\text{or} \quad \frac{dh}{dx} = 0$$

Since $h = \Sigma Y_i h_i$:

$$\frac{dh}{dx} = \frac{d}{dx}(\Sigma Y_i h_i) = \Sigma Y_i \frac{dh_i}{dx} + \Sigma h_i \frac{dY_i}{dx} = 0$$

Apply calorific equation of state: $h_i = h^o_{f,i} + \int c_{p,i}\,dT$

$$\Rightarrow \frac{dh_i}{dx} = 0 + c_{p,i}\frac{dT}{dx}$$

Thus, $$\underbrace{\Sigma Y_i c_{p,i}}_{\equiv c_p} \frac{dT}{dx} + \Sigma h_i \frac{dY_i}{dx} = 0 \qquad (I)$$

To eliminate dY_i/dx, we write species conservation:

PROBLEM 6-8 (continued)

SPECIES CONSV.

$$\dot m_{i x} - \dot m_{i, x+\Delta x} + \dot m_i''' \Delta V = 0$$

Divide by $A \Delta x$, take limit $\Delta x \to 0$, recognize derivative:

$$-\frac{d\dot m_i''}{dx} + \dot m_i''' = 0 \quad ; \text{ where } \dot m_i'' = \dot m'' Y_i \ \& \ \dot m_i''' = \dot\omega_i MW_i$$

Since $\dot m''$ = constant

$$\dot m'' \frac{dY_i}{dx} = \dot m_i''' \quad \text{or} \quad \frac{dY_i}{dx} = \frac{\dot m_i'''}{\dot m''} = \dot\omega_i \frac{MW_i}{\dot m''} \qquad (\mathrm{II})$$

To relate $\dot\omega_i$ to Y_i's we apply the given kinetics:

$$\dot\omega_F = -A \exp(E_a / R_u T)[F][Ox]$$

We can express $[F], [Ox]$ in terms of Y_F, Y_{Ox}:

$$[F] = \frac{P\, MW_{mix}}{R_u T\, MW_F} Y_F \quad ; \quad [Ox] = \frac{P\, MW_{mix}}{R_u T\, MW_{Ox}} Y_{Ox}$$

Since MWs all equal:

$$[F] = \frac{P}{R_u T} Y_F ; \quad [Ox] = \frac{P}{R_u T} Y_{Ox} ; \quad [Pr] = \frac{P}{R_u T} Y_{Pr}$$

Thus

$$\frac{dY_F}{dx} = \frac{MW}{\dot m}\left[-A \exp(E_a / R_u T)\right]\left(\frac{P}{R_u T}\right)^2 Y_F Y_{Ox} \qquad (\mathrm{III})$$

$$\frac{dY_{Ox}}{dx} = \nu \frac{dY_F}{dx} \quad ; \quad \frac{dY_{Pr}}{dx} = -(\nu+1)\frac{dY_F}{dx} \qquad (\mathrm{IV})$$

PROBLEM 6-8 (continued)

Returning to Eqn I,

$$c_p \frac{dT}{dx} + h_F \frac{dY_F}{dx} + h_{ox} \frac{dY_{ox}}{dx} + h_{Pr} \frac{dY_{Pr}}{dx} = 0 \quad (\text{V})$$

Let $Z_1 = -\frac{MW}{\dot{m}''} A \exp(-E_a/R_uT)\left(\frac{P}{R_uT}\right)^2$; Then V becomes;

$$c_p \frac{dT}{dx} + h_F Z_1 Y_F Y_{ox} + h_{ox} \nu Z_1 Y_F Y_{ox} - h_{pr}(\nu+1) Z_1 Y_F Y_{ox} = 0$$

where $h_F = \Delta h_c + c_p(T - T_{ref})$
$h_{ox} = 0 + c_p(T - T_{ref})$
$h_{pr} = 0 + c_p(T - T_{ref})$

Substituting the above, yields

$$c_p \frac{dT}{dx} + Z_1 Y_F Y_{ox} c_p (T - T_{ref}) \left(\cancelto{0}{1 + \nu - (\nu+1)}\right)$$
$$+ Z_1 Y_F Y_{ox} \Delta h_c = 0$$

or

$$c_p \frac{dT}{dx} = -Z_1 Y_F Y_{ox} \Delta h_c \Rightarrow$$

$$\boxed{\dot{m}'' c_p \frac{dT}{dx} = MW\, A \exp(-E_a/R_uT)\left(\frac{P}{R_uT}\right)^2 Y_F Y_{ox} \Delta h_c}$$

COMMENT: We can interpret our final result that the rate-of-change of sensible enthalpy with distance is the "heat-release" associated with combustion.