



AUBURN UNIVERSITY

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AEROSPACE

AERO 4970/7970

Perturbation Methods I
Preliminary Concepts and Gauge Functions

SET I

1. Which of the following is uniformly valid? Explain your reasons.

- a) $e^{\varepsilon x} = 1 + O(\varepsilon)$; $0 \leq x \leq 10$, $\varepsilon \rightarrow 0$
- b) $\frac{1}{x + \varepsilon} = O(1)$; $0 \leq x \leq 1$, $\varepsilon \rightarrow 0$
- c) $\tan(\varepsilon x) = O(\varepsilon)$; $0 \leq x \leq \pi$, $\varepsilon \rightarrow 0$
- d) $e^{-x/\varepsilon} = o(\varepsilon^n)$ for any $n > 0$; $0 \leq x \leq 1$, $\varepsilon \rightarrow 0$

2. Arrange the following in descending order for $\varepsilon \rightarrow 0$. Use l'Hopital's rule if you are uncertain about the ratio of any pair of terms.

$$\varepsilon^{1/2}, \quad \varepsilon e^\varepsilon, \quad e^{-1/\varepsilon}, \quad \varepsilon^{1/2} \ln \varepsilon, \quad \varepsilon \ln^2 \varepsilon.$$

3. Arrange the following in descending order for $\varepsilon \rightarrow 0$. Use l'Hopital's rule if you are uncertain about the ratio of any pair of terms.

$$\ln(1 + \varepsilon), \quad \cot \varepsilon, \quad \tanh(1/\varepsilon), \quad \varepsilon^{-1/2} \sin \varepsilon, \quad 1/\ln \varepsilon$$

4. Determine the first three terms in the Taylor series expansion for the following functions.

- a) $\sqrt{1 - \frac{1}{2}\varepsilon + \varepsilon^2}$
- b) $\sin(\varepsilon - \varepsilon^2)$
- c) $\cos[\frac{1}{4}\pi(1 - \varepsilon x)]$