AUBURN UNIVERSITY

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AEROSPACE

AERO 4970/7970 **Perturbation Methods I** **SET II**

Proper Scaling and Regular Perturbations

1. Nondimensionalize the equation of motion for a mass-spring-damper system $m\ddot{x} + c\dot{x} + kx = 0$. Use $\tilde{x} = x / X$ and $\tilde{t} = t / \tau$, where x and t are the dimensional displacement and time, and $\tilde{x}$ and $\tilde{t}$ are the corresponding dimensionless quantities.
a) If the balance is between the inertia force and the damping force, what must τ equal? What combination of parameters must be small?
b) If the balance is between the spring force and the damping force, what must τ equal? What combination of parameters must be small?

2. Determine a three-term asymptotic expansion for $\operatorname{sech}^{-1}(\varepsilon)$ corresponding to small ε. Plot your results and the exact values for $0 \le \varepsilon \le 1$. A useful piece of information is that

$$\operatorname{sech}^{-1}(x) = \ln\left[\left(1+\sqrt{1-x^2}\right)/x\right]; \quad 0 < x \le 1 .$$

3. When proper assumptions are made, the cooling of a lumped system by combined convection and radiation is governed by the following first order, nonlinear differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}\tau} + \theta + \varepsilon\theta^4 = 0, \text{ subject to the initial condition } \theta(\tau = 0) = 1 .$$

Obtain the three-term asymptotic expansion for the solution and compare it to the exact solution

$$\tau = \tfrac{1}{3}\ln\left[\frac{1+\varepsilon\theta^3}{(1+\varepsilon)\theta^3}\right]$$

by overlaying plots of both.

4. Heat conduction in a one-dimensional slab with a thermal conductivity that varies linearly with temperature is governed by

$$\frac{\mathrm{d}}{\mathrm{d}X}\left[(1+\varepsilon\theta)\frac{\mathrm{d}\theta}{\mathrm{d}X}\right] = 0, \text{ subject to the boundary conditions } \theta(X=0) = 1, \text{ and } \theta(X=1) = 0;$$

where for most engineering materials, $-0.4 \le \varepsilon \le 0.4$. Obtain the first two non-zero terms in the perturbation solution to this problem (i.e., the $\mathcal{O}(\varepsilon^0)$ and $\mathcal{O}(\varepsilon^1)$ terms). Also, solve the equation exactly. Plot both exact and asymptotic solutions for $\varepsilon = -0.3, -0.1, 0.1, 0.3$, when $0 \le X \le 1$.