AUBURN UNIVERSITY

SAMUEL GINN
COLLEGE OF ENGINEERING

AEROSPACE

AERO 4970/7970 **Perturbation Methods I** **SET III**

Transcendental Equations and Successive Approximations

1. Von Karman proposed the following relationship between friction factor, f, and Reynolds number, Re, for turbulent flow in a smooth pipe (with negligible roughness):

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\left(\frac{18.6}{Re\sqrt{f}}\right)$$

This is an inconvenient form from which to obtain friction factors for a given Reynolds number. Develop a two-term asymptotic expression giving friction factors in terms of the Reynolds number. This expression should be valid for large Reynolds numbers corresponding to turbulent flows. Compare your results to the conventional values by plotting your series expansion on a Xerox copy of a Moody diagram.

2. Consider the problem of finding roots of the equation

$$x^4 - 13x^2 + \lambda x + 36 = 0.$$

Obtain the first non-zero term in each of the four solutions for $\lambda \to \infty$ by carefully examining the terms in the equation and finding terms that balance. Why does the former approach, based on the simple regular perturbation expansion in integer powers, give only one solution?

3. Consider the following nonlinear, first-order initial value problem:

$$\frac{\mathrm{d}y}{\mathrm{d}t} + y + \varepsilon y^2 = 0, \quad y(0) = 1.$$

Determine the exact solution and classify this problem as regular or singular in the limit $\varepsilon \to 0$. Classify the problem first for $t > 0$ and then for $t < 0$.

4. When ε is small, determine the three-term expansion for the solution of

$\ddot{u} + u = \varepsilon u^2$, subject to the boundary conditions $u(0) = a$, and $\dot{u}(0) = 0$.

Is the result uniformly valid?