



AUBURN UNIVERSITY

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AEROSPACE

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**AERO 4970/7970**

**Perturbation Methods I  
Method of Strained Coordinates, PLK and Pritulo**

**SET V**

1. The equation of motion for a point mass that slides freely along a parabola ( $x^2 = 2pz$ ) that is rotating about its axis with an angular velocity  $\omega$  is given by

$$(1 + p^{-2}x^2)\ddot{x} + p^{-2}x\dot{x}^2 + (p^{-1}g - \omega^2)x = 0; \text{ with } x(0) = A, \text{ and } \dot{x}(0) = 0.$$

If  $A$  is small, non-dimensionalize the equation and determine a two-term uniformly valid solution using both 1) Pritulo's and 2) Lindstedt's methods. In applying Pritulo's technique, please make use of trigonometric identities.

2. Obtain a two-term uniformly valid solution to the equation:

$$\ddot{u} + u = -\varepsilon u^3; \text{ with } u(0) = a, \text{ and } \dot{u}(0) = 0.$$

3. Use two different methods of strained parameters to obtain a two-term uniformly valid solution to the equation:

$$(x + \varepsilon y)y' - \frac{1}{2}y = 1 + x^2; \text{ with } y(1) = 1.$$

Hint: In using the PLK approach, grouping is necessary before integration.