



AUBURN UNIVERSITY

SAMUEL GINN
COLLEGE OF ENGINEERING

AEROSPACE

AERO 4970/7970

Perturbation Methods I

SET VI

Boundary Layers and Matched Asymptotic Expansions

1. To help understand the concept of the "boundary layer," please plot the exact solution of the equation

$$\varepsilon \ddot{u} + \dot{u} + u = 0; \text{ with } u(0) = 1, \text{ and } u(1) = 2$$

Plot the results for $\varepsilon = 0.1$ and 0.01 . Also, plot the first two terms in the regular perturbation solution using only the outer boundary condition, $u(1) = 2$, the inner solution, and the composite solution. Indicate the region on your plots where the exact and regular perturbation (outer) solutions do not agree. Approximately, what is the thickness of this region?

2. Determine two-term expansions for the inner, outer, and composite solutions of

$$\varepsilon \frac{d^2 y}{dx^2} - \frac{dy}{dx} = 2x; \text{ with } y(0) = \alpha, \text{ and } y(1) = \beta.$$

Where is the boundary layer? Plot the solution please for $\alpha = 1$, $\beta = 2$, $\varepsilon = 0.1$ and 0.01 .