AUBURN UNIVERSITY
SAMUEL GINN COLLEGE OF ENGINEERING
AEROSPACE

AERO 4970/7970 **Perturbation Methods I** **SET VII**

Method of Multiple Scales

1. To help understand the power of multiple scales, please redo the following problem which you have previously analyzed by finding its leading-order multiple-scale solution. This problem was given by

$$\varepsilon\ddot{u} + \dot{u} + u = 0\text{; with } u(0) = 1\text{, and } u(1) = 2$$

For $\varepsilon = 0.1$ and 0.01, compare the one-term multiple scale solution to both exact and two-term composite solutions obtained previously using matched-asymptotic expansions. By comparison to the exact solution, which asymptotic solution appears to be more accurate?

2. Use standard multiple-scale analysis to find the leading-order solution to the following initial value problem valid as $\varepsilon \to 0$.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \varepsilon y^2 \frac{\mathrm{d}y}{\mathrm{d}t} + y = 0\text{; with } y(0) = 1\text{, and } \dot{y}(0) = 0$$

3. Use standard multiple-scale analysis to find the leading-order solution to the following initial value problem valid as $\varepsilon \to 0$. This is an oscillator with cubic damping.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \varepsilon\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^3 + y = 0\text{; with } y(0) = 1\text{, and } \dot{y}(0) = 0$$