Important Series Expansions

Taylor: $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\ldots+\frac{f^n(a)}{n!}(x-a)^n+\ldots$

$(1+x)^\alpha=1+ax+\frac{\alpha(\alpha-1)}{2!}x^2+\frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3+\ldots$

$(1+x)^{-1}=1-x+x^2-x^3+\ldots;\quad -1<x<1$

$(1+x)^{1/2}=1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5x^4}{128}+\frac{7x^5}{256}+\ldots;\quad -1<x<1$

$(1+x)^{-1/2}=1-\frac{x}{2}+\frac{3}{8}x^2-\frac{5x^3}{16}+\frac{35x^4}{128}-\frac{63x^5}{256}+\ldots;\quad -1<x<1$

$\ln x=\frac{x-1}{x}+\frac{1}{2}\left(\frac{x-1}{x}\right)^2+\frac{1}{3}\left(\frac{x-1}{x}\right)^3+\ldots;\quad x\geq\frac{1}{2};$

$\ln x=(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\ldots;\quad |x-1|\leq 1,\quad x\neq 0$

$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots+(-1)^{n+1}\frac{x^n}{n};\quad |x|\leq 1\quad x\neq -1$

$\ln(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\ldots-\frac{x^n}{n};\quad |x|<1$

$\frac{1}{1-x}=1+x+x^2+\ldots+x^n+\frac{x^n}{1-x};\ |x|<1;\quad \frac{1}{(1-x)^2}=1+2x+3x^2+\ldots+nx^n+R;\ |x|<1;\quad \underset{n\to\infty}{R}\to 0$

$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots;\ \sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}\ldots;\ \cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$

$\tan x=x+\frac{x^3}{3}+\frac{2x^5}{15}+\ldots;$ $\qquad \cot x=\frac{1}{x}-\frac{x}{3}-\frac{x^3}{45}-\frac{2x^5}{245}+\ldots$

$\sec x=1+\frac{x^2}{2}+\frac{5x^4}{24}+\frac{61x^6}{720}+\ldots;$ $\qquad \csc x=\frac{1}{x}+\frac{x}{6}+\frac{7}{360}x^3+\frac{31}{15120}x^5+\ldots$

$\sin^{-1}x=x+\frac{x^3}{2.3}+\frac{1\cdot 3\,x^5}{2\cdot 4\cdot 5}+\frac{1\cdot 3\cdot 5\,x^7}{2\cdot 4\cdot 6\cdot 7}+\ldots;\quad |x|<1;\quad \tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots;\quad |x|\leq 1;$

$\tan^{-1}x=\frac{\pi}{2}-\frac{1}{x}+\frac{1}{3x^3}-\frac{1}{5x^5}+\ldots;\quad x>1;$ $\qquad \tan^{-1}x=-\frac{\pi}{2}-\frac{1}{x}+\frac{1}{3x^3}-\frac{1}{5x^5}+\ldots;\quad x<-1$

$\cot^{-1}x=\tan^{-1}(x^{-1});\ \sec^{-1}x=\cos^{-1}(x^{-1});$ $\qquad \csc^{-1}x=\sin^{-1}(x^{-1});\ -1\leq x\leq 1;$

$\sinh x=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots;$ $\qquad \cosh x=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots$

$\tanh x=x-\frac{x^3}{3}+\frac{2x^5}{15}-\frac{17x^7}{315}+\ldots;\quad |x|<\frac{\pi}{2};$ $\qquad \coth x=\frac{1}{x}+\frac{x}{3}-\frac{x^3}{45}+\frac{2x^5}{945}+\ldots$

$\operatorname{sech} x=1-\frac{x^2}{2}+\frac{5x^4}{24}-\frac{61x^6}{720}+\ldots;$ $\qquad \operatorname{csch} x=\frac{1}{x}-\frac{x}{6}+\frac{7x^3}{360}-\frac{31x^5}{15120}+\ldots$

$\sinh^{-1}x=x-\frac{1}{2\cdot 3}x^3+\frac{1\cdot 3}{2\cdot 4\cdot 5}x^5-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7}x^7+\ldots;\quad |x|<1$

$\qquad =\ln 2x+\frac{1}{2\cdot 2\,x^2}-\frac{1\cdot 3}{2\cdot 4\cdot 4\,x^4}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6\,x^6}+\ldots;\quad |x|>1$

$\cosh^{-1}x=\ln 2x-\frac{1}{2\cdot 2\,x^2}-\frac{1\cdot 3}{2\cdot 4\cdot 4\,x^4}-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6\,x^6}+\ldots;\quad |x|>1$

$\tanh^{-1}x=x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\ldots;\quad |x|<1;\quad \coth^{-1}x=\frac{1}{x}+\frac{1}{3x^3}+\frac{1}{5x^5}+\frac{1}{7x^7}+\ldots;\quad |x|>1$

$\cos^3 x=1-\frac{3}{2}x^2+\frac{7}{8}x^4-\frac{x^6}{8}+\ldots;\quad \sin^3 x=x^3-\frac{1}{2}x^5+\frac{3}{4}x^7-\frac{1}{216}x^9+\ldots$