

Useful Trigonometric Identities

FORMULAS FOR ADDITION AND SUBTRACTION

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b; \quad \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b} \quad \cot(a \pm b) = \frac{\cot a \cot b \mp 1}{\cot b \pm \cot a} \quad \tan\left(\frac{\pi}{4} \pm b\right) = \frac{1 \pm \tan b}{1 \mp \tan b} = \frac{\cos b \pm \sin b}{\cos b \mp \sin b}$$

WERNER'S FORMULAS

$$\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]; \quad \sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

INVERSE OF WERNER'S FORMULAS

$$\sin a + \sin b = 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right); \quad \sin a - \sin b = 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)$$

$$\cos a + \cos b = 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right); \quad \cos a - \cos b = -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)$$

OTHER FORMULAS TRANSFORMING SUMS INTO PRODUCTS OF FUNCTIONS

$$\tan a + \tan b = \sin(a+b) / (\cos a \cos b); \quad \tan a - \tan b = \sin(a-b) / (\cos a \cos b)$$

$$\cot a + \cot b = \sin(a+b) / (\sin a \sin b); \quad \cot a - \cot b = -\sin(a-b) / (\sin a \sin b)$$

DUPLICATION FORMULAS

$$\sin 2a = 2 \sin a \cos a; \quad \cos 2a = 2 \cos^2 a - 1 = 1 - 2 \sin^2 a = \cos^2 a - \sin^2 a$$

$$\tan 2a = 2 \tan a / (1 - \tan^2 a); \quad \cot 2a = (\cot^2 a - 1) / (2 \cot a)$$

TRIPLICATION FORMULAS

$$\sin 3a = 3 \sin a - 4 \sin^3 a; \quad \cos 3a = 4 \cos^3 a - 3 \cos a$$

$$\tan 3a = (3 \tan a - \tan^3 a) / (1 - 3 \tan^2 a); \quad \cot 3a = (\cot^3 a - 3 \cot a) / (3 \cot^2 a - 1)$$

BISECTION FORMULAS

$$\sin \frac{a}{2} = \pm \sqrt{\frac{1 - \cos a}{2}}; \quad \cos \frac{a}{2} = \pm \sqrt{\frac{1 + \cos a}{2}}$$

$$\tan \frac{a}{2} = \pm \sqrt{\frac{1 - \cos a}{1 + \cos a}}; \quad \cot \frac{a}{2} = \pm \sqrt{\frac{1 + \cos a}{1 - \cos a}}$$

FORMULAS FOR CONVERTING POWERS INTO MULTIPLE ANGLES

$$\sin^2 a = \frac{1}{2}(1 - \cos 2a); \quad \cos^2 a = \frac{1}{2}(1 + \cos 2a)$$

$$\sin^3 a = \frac{1}{4}(3 \sin a - \sin 3a); \quad \cos^3 a = \frac{1}{4}(3 \cos a + \cos 3a)$$

$$\sin^4 a = \frac{1}{8}(\cos 4a - 4 \cos 2a + 3); \quad \cos^4 a = \frac{1}{8}(\cos 4a + 4 \cos 2a + 3)$$

$$\sin^5 a = \frac{1}{16}(10 \sin a - 5 \sin 3a + \sin 5a); \quad \cos^5 a = \frac{1}{16}(10 \cos a + 5 \cos 3a + \cos 5a)$$

$$\sin^6 a = \frac{1}{32}(10 - 15 \cos 2a + 6 \cos 4a - \cos 6a); \quad \cos^6 a = \frac{1}{32}(10 + 15 \cos 2a + 6 \cos 4a + \cos 6a)$$