



AUBURN UNIVERSITY

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AEROSPACE

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**AERO 4970/7970**

**Advanced Perturbation Methods  
Conventional WKB Analysis**

**SET I**

1. Using the WKB Method, find the first two terms in the solution to the following problem valid as  $\varepsilon \rightarrow 0$ :

$$\varepsilon^2 \frac{d^2 y}{dx^2} + \varepsilon(x-1) \frac{dy}{dx} - xy = 0, \quad y(0) = 1, \quad y(x) \rightarrow 0 \text{ as } x \rightarrow \infty$$

2. Using the WKB Method, find the first three terms in the solution to the following problem valid as  $\lambda \rightarrow \infty$ :

$$\frac{d^2 y}{dx^2} - \lambda^2 xy = 0, \quad y(1) = 1, \quad y(x) \rightarrow 0 \text{ as } x \rightarrow \infty$$

3. Consider the following initial value problem:

$$\delta^2 \frac{d^2 y}{dx^2} - \left( \frac{\kappa^2}{1 - A^2 x^2} \right) y = 0, \quad y(0) = 1, \quad \frac{dy}{dx}(0) = -\frac{\kappa}{\delta A}$$

where  $\delta$ ,  $\kappa$ , and  $A$  are constants, and  $\delta \ll 1$ . Use the WKB Method to generate the first two terms in the solution.

4. Use WKB theory to obtain the first two terms of the solution for  $\varepsilon y'' + a(x)y' + b(x)y = 0$  with  $a(x) > 0$ ,  $y(0) = A$ , and  $y(1) = B$ .