AUBURN UNIVERSITY

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AEROSPACE

AERO 4970/7970 Advanced Perturbation Methods SET III

Boundary Layers

1. Show that $\xi(\eta) = 1 - \exp\left(-\tfrac{1}{2}\eta\kappa / \varepsilon\right)$ represents the Matched-Asymptotic inner solution obtained by solving

$$\varepsilon \frac{\mathrm{d}^2\xi}{\mathrm{d}\eta^2} + \frac{\kappa \sin\eta}{2\eta}\frac{\mathrm{d}\xi}{\mathrm{d}\eta} = 0 \quad \begin{cases} \eta = 0, & \xi = 0 \\ \eta \to \infty, & \xi = 1 \end{cases}$$

Verify your solution by comparing your results, both graphically and tabularally, to the numerical solution of the same second order ODE. Assume that $\varepsilon = 0.001$ and $\kappa = 0.1$. As you set your boundary conditions in your solver (e.g., mathematica or matlab), how do you obtain an accurate solution while avoiding the "division by zero" error at $\eta = 0$. How about the boundary condition at infinity?

2. Given the necessity for a dependent variable transformation, consider the equation

$$\varepsilon\left(\frac{\partial^2 u_z}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial u_z}{\partial \eta}\right) + \frac{\kappa}{2\eta}\sin(\eta)\frac{\partial u_z}{\partial \eta} = -\frac{\pi\kappa^2 z}{\eta}$$

Start by applying the variable transformation $u_z = \xi_z(\eta)\pi z\cos(\eta)$, followed by judicious ordering. Find a suitable equation for $\xi_z(\eta)$.