AUBURN UNIVERSITY

SAMUEL GINN
COLLEGE OF ENGINEERING

AEROSPACE

AERO 4970/7970 Advanced Perturbation Methods SET IV

Rayleigh Janzen Buermann Successive Approximation Error

1. Using a Rayleigh-Janzen expansion series in small M, obtain the first two terms in the solution of:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=\frac{M^2}{2}\left\{\frac{\partial\phi}{\partial x}\frac{\partial}{\partial x}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]+\frac{\partial\phi}{\partial y}\frac{\partial}{\partial y}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]\right.$$
$$\left.+(\gamma-1)\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)\right\} \tag{1}$$

subject to

$$\frac{\partial\phi}{\partial x}(0,y)=0;\quad \frac{\partial\phi}{\partial y}(x,0)=0;\quad \frac{\partial\phi}{\partial y}(x,1)=-1;\quad \phi(0,0)=0. \tag{2}$$

Hint: To solve the partial differential equation, use separation of variables assuming $\phi(x,y)=f(x)+g(y)$.

2. Using Bürmann's theorem and assuming small λ, construct the first three terms in the solution of

$$x^4-13x^2+\lambda x+36=0 \tag{3}$$

Compare your result to the available, regular perturbation expansion (Perturbation I class notes).

3. Consider the transcendental equation for normal shock capture:

$$b\left(\varepsilon\gamma X_0\right)^{(\gamma-1)/(\gamma+1)}-X_0+a\left(\varepsilon\gamma\right)^{(\gamma-1)/(\gamma+1)}X_0^{(2\gamma)/(\gamma+1)}=0 \tag{4}$$

where $\gamma\in[1.1,1.7]$ represents the ratio of specific heats, ε is the area ratio squared, and the two constants a and b, which can be substituted at the very end, are given by

$$a=4^{\frac{\gamma}{\gamma+1}}\frac{1}{\gamma+1};\quad b=\frac{1}{4^{\frac{1}{\gamma+1}}}\frac{\gamma-1}{\gamma(\gamma+1)} \tag{5}$$

a) Show that balancing the first two members of Eq. (4) yields the leading order approximation

$$X_0 = \frac{1}{2}\left[\frac{\gamma - 1}{\gamma(\gamma + 1)}\right]^{\frac{\gamma+1}{2}} (\varepsilon\gamma)^{\frac{\gamma-1}{2}} \tag{6}$$

b) Determine the order of the truncation error associated with X_0.