AUBURN UNIVERSITY

SAMUEL GINN COLLEGE OF ENGINEERING

AEROSPACE

AERO 4970/7970 **Advanced Perturbation Methods** **SET IX**

Adomian Decomposition Method

1. Identify the *linear* $\mathscr{L}$, *remainder* $\mathscr{R}$, and *nonlinear* $\mathscr{N}$ operators in the following differential equations
   a) $\ddot{\theta}+\varepsilon\sin\theta=0; \quad \theta=\theta(t)$
   b) $\ddot{y}+\varepsilon(\dot{y})^3+y=0; \quad y=y(t)$
   c) $xy''''+\varepsilon(yy'''-y'y'')+2y'''=0$

2. Obtain symbolic expressions for the first three Adomian polynomials (A_0, A_1, A_2) for the nonlinear functions of Problem 1 except for Part c) in which you are only required to find A_0 and A_1.

3. Consider the radiation equation

$$T''(x)-\varepsilon T^4(x)=0, \quad T(1)=1; \quad T'(0)=0$$

   a) Idenfity the *linear* $\mathscr{L}$ and *nonlinear* $\mathscr{N}$ operators.
   b) Find the first three Adomian polynomials: $A_0, A_1,$ and A_2.
   c) Solve for the first three terms in the expansion of the Adomian Decomposition Method (ADM): $T_0, T_1,$ and T_2.
   d) Optional: Solve this problem up to second order using a regular perturbation expansion. How does the perturbation solution compare to the ADM solution? Explain.

4. Optional: Consider the transcendental equation

$$e^{x-1}=1+\sqrt{x+1}$$

   This equation admits a root at $x\approx 2.00565$. Use the Adomian Decomposition Method to approximate this root following the steps described below.
   a) Find the first three Adomian polynomials for the function:

$$\mathscr{N}(x)=\ln\left(1+\sqrt{x+1}\right)$$

   Hint: Let $x=x_0+\lambda x_1+\lambda^2 x_2+\ldots$ and use the formula for the Adomian polynomials.
   b) By taking the natural logarithm of the governing equation, identify a nonlinear operator in that equation.
   c) How would you proceed to solve for the root? Hint: by setting x_0 equal to the constant part, then $x_1=A_0, x_2=A_1$, etc.

d) Solve for the root using the first three Adomian polynomials.