Important Vector Identities and Theorems

Below is a compilation of vector identities and theorems written in standard notation, with bold letters representing vectors.

Vector Identities

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = (\mathbf{B}\times\mathbf{C})\cdot\mathbf{A} \quad (1)$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C} \quad (2)$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C}) \quad (3)$$

$$(\mathbf{A}\times\mathbf{B})\times\left(\mathbf{C}\times\mathbf{D}\right) = (\mathbf{A}\times\mathbf{B}\cdot\mathbf{D})\mathbf{C} - (\mathbf{A}\times\mathbf{B}\cdot\mathbf{C})\mathbf{D} \quad (4)$$

$$\nabla(fg) = \nabla(gf) = f\nabla g + g\nabla f \quad (5)$$

$$\nabla\cdot(f\mathbf{A}) = f\nabla\cdot\mathbf{A} + \nabla f\cdot\mathbf{A} \quad (6)$$

$$\nabla\times(f\mathbf{A}) = f\nabla\times\mathbf{A} + \nabla f\times\mathbf{A} \quad (7)$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B}) \quad (8)$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) \quad (9)$$

$$\nabla\cdot(\mathbf{AB}) = (\nabla\cdot\mathbf{A})\mathbf{B} + (\mathbf{A}\cdot\nabla)\mathbf{B} \quad (10)$$

$$\mathbf{A}\times(\nabla\times\mathbf{B}) = (\nabla\mathbf{B})\cdot\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} \quad (11)$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A} + \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) \quad (12)$$

$$\nabla^2 f = \nabla\cdot\nabla f \quad (13)$$

$$\nabla\times(\nabla f) = 0 \quad (14)$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0 \quad (15)$$

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \quad (16)$$

Theorems

$$\int_V \nabla f \,\mathrm{d}V = \int_S \mathbf{n}\cdot f \,\mathrm{d}S \quad (17)$$

$$\int_V \nabla\cdot\mathbf{F}\,\mathrm{d}V = \int_S \mathbf{F}\cdot\mathbf{n}\,\mathrm{d}S \quad (18)$$

$$\int_V \nabla\times\mathbf{F}\,\mathrm{d}V = \int_S \mathbf{F}\times\mathbf{n}\,\mathrm{d}S \quad (19)$$

$$\int_V \left(f\nabla^2 g - g\nabla^2 f\right)\mathrm{d}V = \int_S \mathbf{n}\cdot(f\nabla g - g\nabla f)\,\mathrm{d}S \quad (20)$$

$$\int_V \{\mathbf{A}\cdot[\nabla\times(\nabla\times\mathbf{B})] - \mathbf{B}\cdot[\nabla\times(\nabla\times\mathbf{A})]\}\mathrm{d}V = \int_S \mathbf{n}\cdot[\mathbf{B}\times(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla\times\mathbf{B})]\mathrm{d}S \quad (21)$$

$$\int_V \nabla^2\mathbf{A}\,\mathrm{d}V = \int_S (\hat{\mathbf{n}}\cdot\nabla)\mathbf{A}\,\mathrm{d}S \quad (22)$$

$$\int_V [\mathbf{B}(\nabla\cdot\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B}]\,\mathrm{d}V = \int_S \mathbf{B}(\hat{\mathbf{n}}\cdot\mathbf{A})\,\mathrm{d}S \quad (23)$$