AUBURN UNIVERSITY
SAMUEL GINN
COLLEGE OF ENGINEERING
AEROSPACE

AERO 4970/7970 Fundamentals of Aeroacoustics SET II

Wave Equations, Plane Waves and Fourier Analysis

1. Derive a one-dimensional wave equation for the case when V_0 is not zero (i.e. the air is moving with a uniform velocity in the x direction in the absence of an acoustic wave). Show that the solution is a wave that moves with a velocity $a+V_0$ in the direction of the flow and $a-V_0$ in the direction opposing the flow.

2. Consider the function $p' = e^{i(\omega t - kx)}$:
   a) What is the relationship between ω and k if the function is a solution of the wave equation.
   b) Make a series of sketches of the real part of p' versus x at a fixed time. Consider:
   $t = 0,\ \pi/(2\omega),\ \pi/\omega,\ 3\pi/(2\omega)$, and $2\pi/\omega$.
   c) On the sketches of part b) show the distance (called the wavelength, λ) over which the function goes through one complete cycle. Obtain a relationship between λ and k.
   d) Make a series of sketches showing the real part of the function versus time at:
   $x = \pi/(2k),\ \pi/k,\ 3\pi/(2k)$, and $2\pi/k$. Show the period, T, the time over which the function goes through one cycle, on the sketches.
   e) What distance does the wave travel in one period? How does this compare to the wave length?

3. A rather typical sound wave has a pressure amplitude of $2\text{x}10^{-2}$ Pa and a frequency of 10^3 Hz. Assuming that the wave is a plane wave propagating to the right in air at 22°C and $1.01\text{x}10^5$ Pa:
   a) Write the function describing this wave. Give numerical values for the circular frequency ω, the wave number k, and the amplitude.
   b) What is the amplitude of the particle velocity in this wave?
   c) What is the maximum percentage change in the air density due to this wave?
   d) What is the amplitude of the temperature fluctuations in the wave? (Hint: Substitute the relations $p = p_0 + p'$, $T = T_0 + T'$, and $\rho = \rho_0 + \rho'$ into the gas law and remember that $p_0 = \rho_0 R T_0$)

4. What is the circular frequency, wave length, and period for sound waves with frequencies of 100 Hz, 1,000 Hz, and 10,000 Hz under normal conditions in air?

5. Given the graph of a triangular wave:
   a) Obtain the Fourier series representation of the function shown below.
   b) Using the series obtained write an expression for a wave of this form moving to the right at the speed of sound. Hint: How would you make any one of the terms in the series a wave moving to the right? Try this on all of them.