Inversion of the Stodola Area-Mach Number Relation for the Supersonic Case

This relation provides an estimate for the Mach number at any cross section in the supersonic segment of the nozzle, downstream of the throat; a particular case corresponds to the area $A = A_2$ being in the exit plane.

$$M = \left[\left(\eta\varepsilon^{2\eta}\right)^{\frac{1-k}{2}} - \eta\varepsilon^{2\eta} \right]^{\frac{1}{2\eta}}; \quad \varepsilon = \frac{A_t}{A}; \quad \eta = \frac{k-1}{k+1} \tag{1}$$

Inversion of the Isentropic Flow Relations for Pressure, Temperature and Density

This relation provides an estimate for the pressure, temperature, and density ratios as function of the nozzle area ratio, downstream of the throat; a particular case corresponds to the area $A = A_2$ being in the exit plane. The $n+1$ term expansion is given by

$$s = s_0 + \sum_{m=1}^{n} s_m \; ; \; s_m = \frac{\sum_{i=0}^{m-1} s_i \left(\sum_{i=0}^{m-1} s_i^{\alpha} - \sum_{i=0}^{m-1} s_i^{\beta} - \xi\varepsilon \right)}{\beta \sum_{i=0}^{m-1} s_i^{\beta} - \alpha \sum_{i=0}^{m-1} s_i^{\alpha}} \tag{2}$$

where $\varepsilon = \left(A_t / A\right)^2$, $\xi \equiv \frac{1}{2}\left(k-1\right)\left(\frac{1}{2}k + \frac{1}{2}\right)^{\frac{k+1}{1-k}}$ and

$$s = \begin{cases} \dfrac{p}{p_0} \\ \dfrac{T}{T_0} \\ \dfrac{\rho}{\rho_0} \end{cases} \quad s_0 = \begin{cases} \left(\xi\varepsilon\right)^{\frac{1}{2}k} \\ \left(\xi\varepsilon\right)^{\frac{1}{2}(k-1)} \\ \left(\xi\varepsilon\right)^{\frac{1}{2}} \end{cases} \quad \alpha = \begin{cases} -2/k \\ -2/(k-1) \\ -2 \end{cases}, \; \beta = \begin{cases} -1-1/k \\ -(k+1)/(k-1) \\ -(k+1) \end{cases} \tag{3}$$