Improved solution for the vortical and acoustical mode coupling inside a two-dimensional cavity with porous walls

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This work presents an improved solution to a former study that analyzes the oscillatory motion of gases prescribed by vortico-acoustical mode coupling inside a two-dimensional porous cavity. The physical problem arises in the context of an oscillating gas inside a rectangular enclosure with wall transpiration, sublimation, or sweating. Previously, a multiple-scale solution was derived for the temporal field. The asymptotic formulation was based on an unconventional choice of scales. Its accuracy was also commensurate with the size of ε, a parameter that captured the effect of small viscosity. Currently, an exact solution is derived and compared to the previous formulation. A simple WKBJ solution is also constructed for validation purposes. Unlike both asymptotic formulations, the exact solution remains accurate regardless of the range of physical parameters. [DOI: 10.1121/1.1340648]

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I. INTRODUCTION

The purpose of this work is to provide both exact and asymptotic solutions to a boundary value problem that arises in the context of a fluid oscillating inside a rectangular cavity with transpiring walls. The corresponding mean-flow interactions with oscillatory motion involve time-dependent inertial, convective, and diffusive mechanisms. In a previous article,[1] an approximate solution was obtained using multiple-scale perturbation tools. The perturbation technique used was unconventional in the sense that it relied on a nonlinear scaling transformation. This was necessitated by the intricate inner and outer scaling structures caused by complex interactions of convective, inertial, and diffusive mechanisms. The resulting asymptotic formulation was found to be useful in characterizing the time-dependent field. The latter was a by-product of vortical and acoustical mode interactions that were driven, at the solid boundaries, by the acoustic pressure field. The asymptotic solution found by Majdalani[1] was simple, compact, and practical. However, its accuracy was limited since it depended on the size of a perturbation parameter ε that was the reciprocal of the kinetic Reynolds number. Results were accurate as long as ε remained small. The advantages of removing this limitation were therefore obvious.

Motivated by the desire to seek a more general formulation, an exact analytical solution will be derived here that is independent of ε. Not only will the exact solution be applicable to a wider range of physical parameters, but it will also provide the means to verify the former asymptotic solution in a rigorous fashion. This will be followed by applying a standard WKBJ (Wentzel, Kramers, Brillouin, and Jeffreys) approach to arrive at a leading order, uniformly valid approximation. The WKBJ fomulation will be shown to coincide with the leading order term of the former multiple-scale solution. This will justify the unusual, nonlinear scaling transformation used formerly.[1] Due to its accuracy over a range of flow parameters, the compact multiple-scale formulation will be shown to provide a practical equivalent to the exact solution. Moreover, it will clearly exhibit the functional dependence on physical parameters.

II. MATHEMATICAL ANALYSIS

The mathematical model pertains to the acoustic velocity and pressure fields described by Majdalani.[1] For the sake of consistency, the same geometry and notation will be used here.

A. Velocity field

As detailed in the preceding article,[1] we consider a perfect gas performing small oscillations (at a circular frequency ω_0) about a steady two-dimensional field. This field is established inside a long and narrow cavity of length L, height $2H$, and width W. In a Cartesian reference frame anchored at the cavity's head-end center, y and z represent the cross-flow and streamwise coordinates. All spatial coordinates are normalized by H. For pressure oscillations of amplitude A_p and mean pressure p_0 at the cavity's head end, the total velocity profile can be expressed as a sum of steady and temporal components. The temporal component can be expressed, in turn, as a sum of acoustic, pressure-driven, and solenoidal, vorticity-driven modes. As shown in the preceding article,[1] the total dimensional velocity may be represented by

$$\mathbf{u}^*(y,z,t)=\overbrace{V_b\mathbf{U}(y,z)}^{\text{mean}}+\overbrace{a_0[\hat{\mathbf{u}}(y,z,t)+\tilde{\mathbf{u}}(y,z,t)]}^{\text{unsteady}}, \tag{1}$$

where V_b is the gas velocity at the transpiring wall, a_0 is the speed of sound, and t is dimensionless time. In fact, time $t(=a_0t^*/H)$ is made dimensionless by referring the actual time t^* to the average time it takes for an acoustic pressure disturbance to travel from the porous wall to the centerline, (H/a_0). Note that $\mathbf{u}^*$ is composed of a mean and two time-dependent parts. The dimensionless mean-flow velocity $\mathbf{U}$ is given by

$$\mathbf{U}=U_y\mathbf{e}_y+U_z\mathbf{e}_z=-y\mathbf{e}_y+z\mathbf{e}_z, \tag{2}$$

where $y=y^*/H$ is the dimensionless normal distance measured from the porous wall. Furthermore, $z=z^*/H$ is defined to be the (dimensionless) axial distance measured from the cavity's head-end wall. The dimensionless unit vectors $(\mathbf{e}_y,\mathbf{e}_z)$ have their usual significance. As such, the mean velocity component in Eq. (1) is the product of V_b and the nondimensional spatial vector $\mathbf{U}$.

Having fully defined the steady field inside the cavity, the acoustic velocity that appears in the unsteady part of Eq. (1) can be normalized by the speed of sound a_0 and written as[1]

$$\hat{\mathbf{u}}(z,t)=i\,(\varepsilon_w/\gamma)\,\sin(k_m z)\exp(-ik_m t)\mathbf{e}_z+O(M_b). \quad (3)$$

Here $\varepsilon_w=A_p/p_0$ is the pressure wave amplitude, γ is the ratio of specific heats, $k_m=m\pi H/L$ is the dimensionless wave number, and $m=1,2,3,...$ is the acoustic mode number. Note that these parameters are all unitless.

The second unsteady part of Eq. (1) represents the vortical velocity response. This component stems from the linearized, rotational momentum equation known to the order of the cross-flow Mach number $(M_b=V_b/a_0\ll 1)$. This can be expressed by

$$\tilde{\mathbf{u}}(y,z,t)=\mathbf{V}(y,z)\exp(-ik_m t), \quad \mathbf{V}(y,z)=V_y\mathbf{e}_y+V_z\mathbf{e}_z. \quad (4)$$

Noting that $V_y/V_z=O(M_b)$, the corresponding vortical mass and momentum conservation equations are

$$\partial V_y/\partial y+\partial V_z/\partial z=0, \quad (5)$$

$$iV_z=\sigma\left[\frac{\partial}{\partial z}(V_z U_z)+U_y\frac{\partial V_z}{\partial y}\right]-\varepsilon\frac{\partial^2 V_z}{\partial y^2}+O(M_b), \quad (6)$$

where

$$\varepsilon\equiv 1/\mathrm{Re}_k=\nu_0/\omega_0 H^2, \quad \sigma\equiv 1/\mathrm{Sr}=V_b/\omega_0 H. \quad (7)$$

The last two parameters represent the reciprocals of the kinetic Reynolds number, Re_k, and the Strouhal number, Sr. Since, in practice, $\mathrm{Re}_k>10^3$ and $\mathrm{Sr}>10$, both ε and σ can be used as primary and secondary perturbation parameters. Note that the Strouhal number is the product of the circular frequency and characteristic height $(\omega_0 H)$ divided by the velocity at the boundary V_b. Since V_b is two-to-three orders of magnitude smaller than a_0, Sr is much larger than the familiar aeroacoustic Strouhal number based on the speed of sound. The latter extends over the range $[10^{-3},10]$ with a peak in the noise spectrum occurring at $\mathrm{Sr}\approx 0.2$. In the current analysis, Sr extends over the range $[10,10^3]$ with a commonly reported value of $\mathrm{Sr}\approx 50$.

When separation of variables is applied to the momentum equation (6), the vortical velocity component can be solved for. The result is

$$V_z(y,z)=-\frac{\varepsilon_w}{\gamma}i\sum_{n=0}^{\infty}\frac{(-1)^n(k_m z)^{2n+1}}{(2n+1)!}Y_n(y). \quad (8)$$

Here the velocity eigenfunction $Y_n(y)$ must be determined from the boundary-value problem defined by

$$\varepsilon\frac{d^2Y_n}{dy^2}+\sigma y\frac{dY_n}{dy}+[i-(\lambda_n+1)\sigma]Y_n=0, \quad \lambda_n=2n+1, \quad (9)$$

and

$$Y_n(1)=1 \quad \text{(no-slip)}, \quad (10)$$

$$dY_n(0)/dy=0 \quad \text{(centerline symmetry)}. \quad (11)$$

B. Unconventional multiple-scale solution

Using two fictitious scales, $y_0=y$ and $y_1=\varepsilon y^{-2}$, a multiple-scale solution for Y_n was derived before by Majdalani.[1] Using the superscript M for "multiple scale," this solution is reproduced here for convenience and added clarity

$$Y_n^M(y)=y^{2n+2}\exp\{-\xi[1-\sigma^2(2n+1)(2n+2)]\eta - i[\ln y+\xi\sigma^2(4n+3)\eta]/\sigma\}+O(\varepsilon), \quad (12)$$

where $\xi=\mathrm{Sr}^3/\mathrm{Re}_k$ is a viscosity parameter. Note that the asymptotic solution strongly depends on

$$\eta(y)=\int_1^y U_y^{-3}(\tau)\,d\tau=(y^{-2}-1)/2. \quad (13)$$

Physically, η represents the characteristic length scale normal to the porous wall. It thus controls the rate of decay of the rotational wave amplitude as the distance from the porous wall is increased. For example, near the porous wall, $\eta(1)=O(1)$, and so is the rotational wavelength. However, as the core is approached, $\eta(0)\to\infty$, and the rotational wavelength becomes infinitesimally small.

At this point, one may substitute Eq. (12) back into Eqs. (8) and (4). At the outset, one obtains

$$\tilde{u}_z^M(y,z,t)=-(\varepsilon_w/\gamma)\,iy\,\sin(k_m yz)\exp[-(1-2\sigma^2)\xi\eta - i(\ln y+3\xi\sigma^2\eta)/\sigma-ik_m t]+O(\varepsilon). \quad (14)$$

From continuity, the normal component $\tilde{u}_y$ can be determined. Thus using $\partial\tilde{u}_y/\partial y=-\partial\tilde{u}_z/\partial z$, one gets

$$\tilde{u}_y^M(y,z,t)=-(\varepsilon_w/\gamma)\,M_b y^3\cos(k_m yz)\exp[-(1-2\sigma^2)\xi\eta - i(\ln y+3\xi\sigma^2\eta)/\sigma-ik_m t]+O(\varepsilon), \quad (15)$$

where $\tilde{u}_y/\tilde{u}_z=O(M_b)$ is confirmed. Superimposing acoustic and vortical fields gives the total oscillatory velocity. Writing at $O(M_b)$ renders, at length,

$$u_z^M(y,z,t)=(\varepsilon_w/\gamma)\,i\exp(-ik_m t)\{\sin(k_m z)-y\sin(k_m yz)\times\exp[-(1-2\sigma^2)\xi\eta-i(\ln y+3\xi\sigma^2\eta)/\sigma]\}, \quad (16)$$

whose real part is

$$u_z^M(y,z,t)=\frac{\varepsilon_w}{\gamma}\Bigg[\overbrace{\sin(k_m z)\sin(k_m t)}^{\text{acoustic part}} \overbrace{-\underbrace{y\sin(k_m yz)\exp(-\zeta)}_{\text{vortical amplitude}}\underbrace{\sin(k_m t+\Phi)}_{\text{wave propagation}}}^{\text{vortical part}}\Bigg], \quad (17)$$

where

$$\zeta = \xi(1-2\sigma^2)\,\eta, \quad \Phi = (\ln y + 3\xi\sigma^2\eta)/\sigma. \tag{18}$$

C. Exact solution

The separated momentum equation (9) can be expressed as:

$$R^{-1}Y_n'' + yY_n' + [i\,\mathrm{Sr} - (2n+2)]Y_n = 0; \qquad R \equiv \mathrm{Re}_k/\mathrm{Sr} = V_b H/\nu_0. \tag{19}$$

In addition to Sr, Eq. (19) appears to be controlled by the cross-flow Reynolds number R.

1. Liouville–Green transformation

An exact solution to Eq. (9) seems possible if a transformation can be implemented in a manner to eliminate all variable coefficients from the differential equation. To that end, a Liouville–Green transformation may be attempted (cf. Nayfeh,[2] pp. 364–366). This requires setting

$$X = \phi(y), \quad F(X) = \psi(y)Y_n(y), \tag{20}$$

where $\phi(y)$ and $\psi(y)$ must be determined in a manner to transform Eq. (19) into a simpler equation in $F(X)$ that exhibits an exact solution. Starting with Eq. (20), derivatives become

$$Y_n' = -\frac{\psi'}{\psi^2}F + \frac{1}{\psi}\frac{dF}{dX}\frac{dX}{dy} = -\frac{\psi'}{\psi^2}F + \frac{\phi'}{\psi}\frac{dF}{dX}, \tag{21}$$

$$Y_n'' = \frac{\phi'^2}{\psi}\frac{d^2F}{dX^2} + \left(\frac{\phi''}{\psi} - \frac{2\phi'\psi'}{\psi^2}\right)\frac{dF}{dX} - \left(\frac{\psi''}{\psi^2} - \frac{2\psi'^2}{\psi^3}\right)F. \tag{22}$$

Substitution into Eq. (19) gives

$$\frac{d^2F}{dX^2} + \frac{1}{\phi'^2}\left(\phi'' - \frac{2\phi'\psi'}{\psi} + yR\phi'\right)\frac{dF}{dX} + \frac{1}{\phi'^2}\left\{\left(-\frac{\psi''}{\psi} + \frac{2\psi'^2}{\psi^2} - \frac{yR\psi'}{\psi}\right) + R[i\,\mathrm{Sr} - (2n+2)]\right\}F = 0. \tag{23}$$

At this juncture, functions ϕ and ψ must be chosen so that dominant parts of the transformed equation have constant coefficients. For example, the coefficient of dF/dX can be forced to be zero by setting

$$\phi'' - \frac{2\phi'\psi'}{\psi} + yR\phi' = 0, \ \text{ or } \ \frac{\psi'}{\psi} = \frac{\phi''}{2\phi'} + \frac{yR}{2}. \tag{24}$$

This algebraic maneuver leads to an expression for ψ that transforms Eq. (23) into an equation that can be solved exactly. As a matter of fact, direct integration of Eq. (24) gives $\psi = K_0\sqrt{\phi'}\exp(Ry^2/4)$, where K_0 is an arbitrary constant. This reduces Eq. (23) into

$$d^2F/dX^2 + \{(R/\phi'^2)[i\,\mathrm{Sr} - (2n+2)] + \delta\}F = 0, \qquad \delta = (1/\phi'^2)(-\psi''/\psi + 2\psi'^2/\psi^2 - yR\psi'/\psi). \tag{25}$$

The first derivative is hence eliminated. Next, one imposes

$$(R/\phi'^2)[i\,\mathrm{Sr} - (2n+2)] = \mathrm{Const}, \tag{26}$$

so that $\phi' = \sqrt{R}$. As one sets $K_0 = R^{-1/4}$, $X = \phi(y) = y\sqrt{R}$, and $\psi(y) = \exp(Ry^2/4)$. The transformed equation becomes

$$\frac{d^2F}{dX^2} + (p + \tfrac{1}{2} - \tfrac{1}{4}X^2)F = 0, \quad p \equiv -3 - 2n + i\,\mathrm{Sr}. \tag{27}$$

Likewise, the two physical boundary conditions given by Eqs. (10) and (11) translate into

$$F(\sqrt{R}) = \exp(R/4), \quad dF(0)/dX = 0. \tag{28}$$

2. Exact solution

Equation (27) possesses a standard solution that is expressible in terms of the parabolic cylinder function $D_p(X)$:

$$F(X) = C_1 D_p(X) + C_2 D_p(-X). \tag{29}$$

Since $\mathrm{Re}(p) < 0$, formula 9.241.2 in Gradshtein and Ryzhik[3] (cf. p. 1092) can be used for $D_p(X)$. Accordingly,

$$D_p(X) = [\Gamma(-p)]^{-1}\exp(-\tfrac{1}{4}X^2) \times \int_0^\infty \tau^{-p-1}\exp(-\tau X - \tfrac{1}{2}\tau^2)\,d\tau, \tag{30}$$

where Γ is Euler's Integral of the second kind. Careful application of boundary conditions gives, after some effort,

$$F'(0) = -2^{-1/2(1+p)}\Gamma[\tfrac{1}{2}(1-p)](C_1 - C_2)/\Gamma(-p) = 0, \tag{31}$$

$$C_1 = C_2 = 2^{p/2}\exp(\tfrac{1}{2}R)\Gamma(-p)/[\Gamma(-\tfrac{1}{2}p)\Phi(-\tfrac{1}{2}p, \tfrac{1}{2}, \tfrac{1}{2}R)], \tag{32}$$

where Φ is the confluent hypergeometric function given by

$$\Phi(a,b;x) = 1 + \frac{a}{b}\frac{x}{1!} + \frac{a(a+1)}{b(b+1)}\frac{x^2}{2!} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)}\frac{x^3}{3!} + \cdots. \tag{33}$$

Using the superscript E for ''exact,'' we write

$$Y_n^E(y) = \exp[\tfrac{1}{2}R(1-y^2)]\Phi(-\tfrac{1}{2}p, \tfrac{1}{2}, \tfrac{1}{2}Ry^2)/\Phi(-\tfrac{1}{2}p, \tfrac{1}{2}, \tfrac{1}{2}R). \tag{34}$$

The remaining equations follow precisely Eqs. (14)–(16) via Eq. (8). For example,

$$\tilde{u}_z^E(y,z) = -\frac{\varepsilon_w}{\gamma}i\exp(Ry^2\eta - ik_m t) \times \sum_{n=0}^{\infty}\frac{(-1)^n(k_m z)^{2n+1}\Phi(-\tfrac{1}{2}p, \tfrac{1}{2}, \tfrac{1}{2}Ry^2)}{(2n+1)!\,\Phi(-\tfrac{1}{2}p, \tfrac{1}{2}, \tfrac{1}{2}R)}, \tag{35}$$

and

TABLE I. Exact and asymptotic predictions for $\mathrm{Sr}=50$, $\mathrm{Re}_k=10^6$, and $n=0$.

y	Y_n^E Eq. (34) Exact sol.	Y_n^M Eq. (12) Multi-scale	Y_n^W Eq. (40) WKBJ sol.	$\left\|\frac{Y_n^E-Y_n^M}{Y_n^E}\right\|$ %	$\left\|\frac{Y_n^E-Y_n^W}{Y_n^E}\right\|$ %
0.25	0.024 493 5	0.024 242 2	0.023 989 0	1.0300	2.0600
0.30	−0.043 164 5	−0.042 664 3	−0.041 790 0	1.1600	3.1800
0.35	−0.045 410 8	−0.046 011 2	−0.047 679 7	1.3200	5.0000
0.40	−0.026 949 6	−0.027 597 5	−0.029 787 6	2.4000	10.500
0.45	−0.094 223 1	−0.094 654 6	−0.096 500 0	0.4580	2.4200
0.50	−0.206 560 1	−0.206 475 5	−0.206 225 2	0.0410	0.1620
0.55	0.009 566 5	0.009 970 2	0.012 231 6	4.2200	27.900
0.60	0.296 660 9	0.296 503 4	0.295 630 4	0.0531	0.3470
0.65	−0.347 997 5	−0.348 100 1	−0.348 949 3	0.0295	0.2740
0.70	0.240 239 7	0.240 423 3	0.241 935 7	0.0764	0.7060
0.75	−0.129 288 0	−0.129 459 7	−0.130 970 6	0.1330	1.3000
0.80	0.097 999 9	0.098 135 3	0.099 419 1	0.1380	1.4500
0.85	−0.188 407 6	−0.188 502 6	−0.189 477 8	0.0504	0.5680
0.90	0.420 399 6	0.420 452 5	0.421 044 2	0.0126	0.1530
0.95	−0.751 123 7	−0.751 139 1	−0.751 333 2	0.0020	0.0279
1.00	1.000 000 0	1.000 000 0	1.000 000 0	0.0000	0.0000

$$u_z^E(y,z,t)=\frac{\varepsilon_w}{\gamma}i\exp(-ik_mt)\left[\sin(k_mz)-\exp(Ry^2\eta)\times\sum_{n=0}^{\infty}\frac{(-1)^n(k_mz)^{2n+1}\Phi(-\frac{1}{2}p,\frac{1}{2},\frac{1}{2}Ry^2)}{(2n+1)!\Phi(-\frac{1}{2}p,\frac{1}{2},\frac{1}{2}R)}\right].\quad(36)$$

The real part of u_z^E may now be compared to the multiple-scale solution given by Eq. (17).

D. Standard WKBJ solution

In seeking a standard asymptotic solution for Eq. (9), it should be noted that two cases must be considered depending on the order of the Stouhal number.

1. Standard outer expansion

For $\mathrm{Sr}=O(1)$, $y=O(1)$, and one must resolve, at leading order, the outer solution Y_n^o from

$$\sigma yY_n^{o\prime}+[i-(2n+2)\sigma]Y_n^o=0,$$
$$Y_n^o(1)=1,\ \text{or}\ Y_n^o(y)=y^{2n+2}\exp(-i\,\mathrm{Sr}\ln y).\quad(37)$$

On the one hand, the y^{2n+2} factor in Y_n^o decays rapidly as $y\to0$. As a result, the remaining boundary condition at the origin is automatically satisfied by the first derivative. This obviates the need for an inner solution of this order. On the other hand, the exponential term in Y_n^o represents an oscillatory behavior that is rapid for $\mathrm{Sr}>10$. Since Sr can be large in practice, rapid oscillations that occur on a shorter scale preclude the possibility of a uniformly valid solution. This becomes apparent in the expression for the first order correction when the outer solution is written at $O(\varepsilon^2)$:

$$Y_n^o(y)=y^{2n+2}\exp(-i\,\mathrm{Sr}\ln y)\{1+\varepsilon\,\mathrm{Sr}[-\mathrm{Sr}^2+2n(2n+1)-i(4n+1)\mathrm{Sr}](y^{-2}-1)/2\}.\quad(38)$$

In fact, since the correction term comprises a part of $O(\varepsilon\,\mathrm{Sr}^3)$, nonuniformity can be expected for large Sr. At the outset, a WKBJ expansion may be called upon.

2. The WKBJ expression

For $\mathrm{Sr}>10$, rapid oscillations occur on a short scale, and there is a slow drift on the scale $y=O(1)$. The leading order equation that provides the WKBJ ansatz is

$$yY_n'+i\,\mathrm{Sr}\,Y_n=0,\quad Y_n(1)=1,\ \text{or}\ Y_n(y)=\exp(-i\,\mathrm{Sr}\ln y).\quad(39)$$

This suggests posing $Y_n^W(y)=g(y)\exp(-i\,\mathrm{Sr}\ln y)$ and substituting back into Eq. (9). The emerging formulation can be obtained at $O(\varepsilon\,\mathrm{Sr}^2)$:

$$Y_n^W(y)=y^{2n+2}\exp(-\xi\eta-i\,\mathrm{Sr}\ln y).\quad(40)$$

Clearly, the standard WKBJ solution Y_n^W matches the leading order term of Eq. (12). This confirms the validity of the former multiple-scale solution Y_n^M. Note that both asymptotic and exact formulations depend exponentially on the function η.

E. Comparison

For $\mathrm{Re}_k=10^6$ and $\mathrm{Sr}=50$, exact, multiple-scale, and WKBJ predictions are displayed in Table I. Included are the percentage deviations of asymptotic solutions from Y_n^E. In some entries, it is gratifying to note that Y_n^M and Y_n^W match Y_n^E in several decimal places. This agreement is typical. Naturally, the accuracy of asymptotic predictions deteriorates for smaller values of Re_k.

III. CONCLUSIONS

This work extends the preceding article[1] by presenting an exact solution for the temporal field inside a porous cavity. The exact solution serves a dual purpose. Despite its relative complexity by comparison to the asymptotic formulations, it provides accurate predictions over a far broader range of physical parameters. It also serves as a benchmark for validating other possible, approximate solutions. For example, its exponentially decaying argument is shown to depend on a characteristic length scale that also appears in the asymptotic formulations. This spatial scale can be ascribed to the nonlinear scaling constitution arising in this problem. It therefore explains the need for an unconventional scaling transformation in the multiple-scale expansion used in the preceding article.[1] The standard WKBJ analysis introduced here is also confirmatory. In hindsight, the establishment of an exact solution for the problem at hand does not undermine the usefulness of asymptotic formulations. The latter have the advantage of being expressible in simple finite forms that clearly display the dependence on physical agents. They hence remain quite practical over a substantial range of parameters corresponding to a Strouhal number in excess of 10 and a kinetic Reynolds number in excess of 1000.

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