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### **Premixed Flame Response to Acoustic** Waves in a Porous-Walled Chamber with **Surface Mass Injection**

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In order to study the internal coupling between flame dynamics and vortico-acoustic waves formed during solid propellant combustion, a numerical simulation of an idealized rocket combustion chamber is carried out. The chamber is modeled as a rectangular enclosure along whose porous walls a laminar mixture of premixed reactants is uniformly injected. The mathematical model is based on the conservation equations in two space dimensions. Full account is taken of variable thermo-physical properties and finite-rate chemical kinetics. Boundary conditions are specified using the method of characteristics that accommodates the transport of entropy, vorticity, and acoustic waves. The governing equations and associated boundary conditions are solved numerically using a preconditioning technique and a dual time-stepping integration procedure. For illustrative purposes, a propane-air mixture is injected through the chamber walls in an effort to emulate the flame evolution inside a solid-propellant rocket motor under laminar conditions. First, detailed flame structures under steady-state conditions are realized. Subsequently, traveling acoustic waves are imposed at the head end. Simulation results indicate that the oscillatory velocity exhibits a multi-dimensional structure caused by unsteady vorticity, pressure, and flame oscillations. Accordingly, the effects of laminar premixed flame oscillations are limited to a thin region above the burning surface. This region becomes thinner as the frequency of oscillations is increased. The flowfield outside the flame zone bears a striking resemblance to recent analytical solutions obtained in a geometrically similar chamber.

	Nomenclature	Q	rate of heat release
a	speed of sound $\sqrt{\gamma RT}$	ġ	rate of heat release per unit mass
A	pressure amplitude	R	the ideal gas constant for reactants
$\hat{C}$	specific heat	Re	injection Reynolds number
$\mathbf{E}_{p}$	convective flux vectors	Т	temperature
L,I F F	diffusive flux vectors	t	physical time
$\boldsymbol{L}_{\mathrm{v}}, \boldsymbol{\Gamma}_{\mathrm{v}}$	specific total energy	u, v	velocities in x and y directions, respectively
e k	thermal conductivity	V	specific volume
л М	molecular weight of species <i>i</i>	$V_{h}$	blowing velocity in a cold-flow model
NI <sub>i</sub>	pressure	v,	transverse velocity directly above the flame
p	pressure	<i>v</i>	transverse velocity directly above the wall
		x. v	Cartesian coordinates
*Graduate Research Associate. Member AIAA.		$Y_i$	mass fraction of species <i>i</i>
<sup>†</sup> Profess	or. Department of Mechanical Engineering, Fellow	-1	muss muction of species /
AIAA.		Greek Symbols	
$\frac{1}{2}$ Graduate Research Associate Member AIAA		Sieck	thickness of the reactive flame zone
Oraduate Research Associate. Melliber AIAA.		$O_f$	unexhess of the reactive fiame zone.

 $\partial_f$ λ spatial wavelength of unsteady velocity

ratio of specific heats γ

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- $\mu$ ,  $\nu$  dynamic and kinematic viscosities
- $au_c$  characteristic time associated with heat release fluctuation
- $\dot{\omega}_i$  mass production rate of species *i*
- **Ω** vorticity

#### Superscripts

- ' acoustic fluctuation
- steady state
- ^ amplitude of fluctuating quantity

#### Subscripts

- *a* acoustic mode
- *b* blowing condition for a cold flow
- f flame condition
- *i* chemical species *i*
- s entropy mode
- v vorticity mode
- w wall condition

#### I. Introduction

**T**N this article, the premixed laminar flame response to L vortico-acoustic oscillations is investigated inside a porous-walled chamber simulating a solid rocket motor. Following the injection of a premixed flow of reactants across the chamber walls, the resulting thin laminar flame that forms in the vicinity of the porous wall is used to mimic the burning zone in solid propellant rocket motors. Harmonic pressure fluctuations are superimposed to represent acoustic oscillations in actual motors. This work focuses on characterizing the thermo-acoustic boundary layer above the injection surface, which plays an important role in determining the energy exchange between unsteady combustion processes and core flow dynamics.<sup>1</sup> Understanding the mechanisms behind these thermo-acoustic energy exchange processes helps to improve our ability to predict combustion instabilities in rocket motors, gas turbines, and other large combustors.

Combustion instabilities arise in a number of physical contexts that generally exhibit a self-sustained coupling between unsteady chemical heat sources and chamber acoustics. Understanding their attributes is of crucial importance in the development of combustion devices such as solid rocket motors,<sup>2</sup> liquid rocket engines,<sup>3,4</sup> gas turbines,<sup>5</sup> and pulsed combustors of the Helmholtz<sup>6</sup> and Rijke types.<sup>7</sup>

Initially, small-amplitude acoustic disturbances arise from the intrinsic flow instabilities in the chamber. If these disturbances oscillate in favorable phase with the rate of heat addition, they are amplified in accordance with Rayleigh's criterion. In certain applications, the growing instability may reach a level sufficient to cause undesirable structural vibrations or outright failure of the system. A thorough understanding of this energy transfer mechanism requires detailed studies of a) *fluid dynamic processes* that dictate the environment in which chemical reactions occur, and b) *chemical processes* that provide the energy for driving thermoacoustic instabilities. Clearly, the interaction between these two processes plays a key role in determining the dynamic behavior of unsteady motions in combustion chambers.

The present paper deals with the unsteady response of a laminar flame to traveling acoustic waves in a twodimensional chamber. A simple geometry is selected in order to facilitate theoretical and numerical comparisons. Unlike other studies that have focused on turbulent flames in similar physical settings,<sup>8-12</sup> the scope here will be limited to laminar flames only. The exploration of laminar flames will be shown to facilitate the identification of key mechanisms that can be corroborated by theoretical solutions.

The study begins in Sec. II by a description of the model and its attendant boundary conditions. The numerical scheme is briefly visited in Sec. III where it is followed by a steady-state solution of the problem at hand. After describing the mean flow and thermal characteristics in Sec. IV, the onset of unsteady disturbances is examined in Sec. V. Therein, the thermo-acoustic coupling due to heat release is explored at different frequencies and spatial locations along the length of the chamber. The Rayleigh number is also invoked to explain the connection between vorticoacoustic waves and thermal oscillations. Finally, conclusions are provided in Sec. VI.

#### **II.** Formulation

#### A. Model

The planar domain is shown schematically in Fig. 1a. The chamber is closed at the head-end, while the porous top and bottom surfaces admit a premixed combustible mixture. The injection process simulates the gas-phase combustion and flow development in a rocket motor loaded with a homogeneous solid propellant. A similar configuration was experimentally studied by Sankar et  $al^{2}$  Here we shall study the mutual coupling between flow oscillations and flame dynamics, with emphasis placed on the interactions among the acoustic, vortical, and entropy waves that arise in the chamber. But first, a numerical analysis based on the complete conservation equations and finite-rate chemical kinetics will be developed. At the outset, various fundamental mechanisms, including vorticity generation and transport, flame oscillations, and acoustic motions, will be investigated.

#### **B.** Equations

The formulation is based on the conservation equations of mass, momentum, energy, and species concentrations for a multi-component, chemically reacting system. Full account is taken of finite-rate chemical kinetics and variations in thermo-dynamical and transport properties. To start, the governing equations in two-dimensional Cartesian coordinates are written in the generic vector form

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \frac{\partial}{\partial x} (\boldsymbol{E} - \boldsymbol{E}_{v}) + \frac{\partial}{\partial y} (\boldsymbol{F} - \boldsymbol{F}_{v}) = \boldsymbol{S} \quad (1)$$

where x and y represent the axial and transverse coordinates, respectively. Using standard descriptors, the vector of conserved variables Q is defined by

$$\boldsymbol{\mathcal{Q}} = \begin{bmatrix} \rho, \ \rho u, \ \rho v, \ \rho e, \ \rho Y_i \end{bmatrix}^T$$
(2)

The inviscid flux vectors ( $\boldsymbol{E}$ ,  $\boldsymbol{F}$ ) and the viscous flux vectors ( $\boldsymbol{E}_v$ ,  $\boldsymbol{F}_v$ ) follow the definitions given by Chu.<sup>13</sup> The thermal and mass diffusion processes are approximated by Fourier's and Fick's laws, respectively. The source term  $\boldsymbol{S}$  includes contributions from chemical reactions taking place inside the combustion chamber.

Within the temperature and pressure ranges of the present study, the specific heat, viscosity, and thermal conductivity are taken to be functions of temperature and can be approximated by fourth-order polynomials.<sup>14</sup> The mixture specific heat is obtained by mass-fraction weighing of each constituent species. Thermal conductivity and viscosity are calculated using Wilke's mixing rule. The binary mass diffusivity  $D_{ij}$  between species *i* and *j* is obtained using the Chapman-Enskog theory based on the Lennard-Jones intermolecular potential-energy functions.<sup>15</sup> The effective mass diffusion coefficient for species *i* in a multi-component mixture is related to the binary diffusion coefficients through Wilke's formula.<sup>16</sup>

Since the present study is concerned with the effect of unsteady heat release and flow motion on chamber aeroacoustics, an accurate estimate of the flame speed and heat release rate is essential. The single-step global reaction mechanism proposed by Westbrook and Dryer<sup>17</sup> for hydrocarbon/air systems is therefore adopted because of its reasonable prediction of these two parameters. The small number of species involved further renders this model an attractive choice from the standpoint of computational efficiency.

#### C. Boundary Conditions

The method of characteristics is used to handle the boundary conditions. Since the flow is subsonic throughout the chamber, three physical conditions are specified at the inflow boundary and only one at the



Fig. 1 Schematic diagram of the physical model illustrating: a) the adopted space coordinates, flame zone location, and mean velocity evolution across the flame zone; and b) the non-uniform grid extending from the wall to the core.

exit. The mass flow rate and temperature of the premixed gases are fixed at the porous walls, along with the no-slip condition that enforces normal injection of the reactants. At the head end, the axial velocity and pressure gradient, as well as the radial velocity gradient, are set to zero for steady-state calculations. This prevents the occurrence of numerically generated recirculation near the head end. It also coincides with the head end boundary conditions used in mathematical idealizations of non-reactive core-flows.<sup>18</sup> For unsteady flow simulations, a periodic sinusoidal pressure oscillation is imposed at a known frequency. The axial velocity and temperature fluctuations are accordingly specified from the isentropic relations to generate a traveling acoustic wave propagating in the axial direction. Finally, flow symmetry is assumed along the midsection plane.

The exit boundary conditions are also treated carefully to avoid the production of numerically generated, spurious reflecting waves. For this purpose, the methodology described by Watson and Meyer<sup>19</sup> is used to specify the flow properties at the exit section. The governing equations are rearranged into four decoupled relations controlling the vorticity, entropy, and acoustic wave propagation. The complete form of these relations are imposed implicitly to be consistent with the treatment of the interior points.

#### III. Numerical Method

The solution algorithm is based on a preconditioning technique developed to circumvent the computational difficulties arising in low Mach-number reacting flows.<sup>20,21</sup> Temporal discretization is obtained using a second-order dual-time stepping integration, and a second-order flux differencing technique for spatial discretization. The governing equations are treated in a fully coupled manner by means of an alternating-direction-implicit (ADI) factorization scheme.<sup>22</sup> The implicit treatment and dual-time stepping procedure guarantees a converged solution at every physical time step. This procedure also helps in capturing the transients in the flowfield and its attendant chemical reactions.

The numerical scheme described above is implemented to study the flame response to traveling acoustic waves in a porous chamber. The chamber has an effective aspect ratio of 20, being 1 m in length (*L*) and 0.05 m in wall-to-core height (*H*). A propane-air mixture having an equivalence ratio of 0.7 is injected across the porous walls at a flow rate of 0.21 kg m<sup>-2</sup>s<sup>-1</sup>, a temperature ( $T_w$ ) of 350 K, and a pressure (*P*) of 1 bar. The flow rate translates into an injection velocity  $(v_w)$  of 0.206 ms<sup>-1</sup> and a surface Mach number  $(M_w)$  of  $5.6 \times 10^{-4}$  where the speed of sound  $(a_w)$  is 368 m/s. After crossing the reactive flame zone  $(y = \delta_f)$ , the gas velocity  $(v_f)$  increases to a speed of 1.154 ms<sup>-1</sup>, at a Mach number  $(M_f)$  of  $1.36 \times 10^{-3}$  (see Fig. 1a). The computational grid is illustrated in Fig. 1b and consists of 60 by 100 non-uniform cells in the axial and transverse directions, respectively. In order to better resolve the rapid variations of flow properties in the flame zone, cells are clustered more closely near the porous boundary. The smallest cell is nearly 0.015 mm-thick just above the transpiring wall.

#### **IV. Steady-State Results**

Before addressing the characteristics of the unsteady flow, it is helpful to examine the simulation results for the mean flowfield. Figure 2 illustrates the distributions of mean flow Mach number, vorticity, and gauge pressure under steady-state conditions. Because the configuration is symmetric, only the lower half of the chamber is displayed.

#### A. Mean Velocity and Pressure Distributions

As indicated in Fig. 2, the coordinates y/H = 0 and 1 correspond to the porous wall and core, respectively. Throughout, Mach-number contours are shown to exhibit the two-dimensional structure typical of an



Fig. 2 Contours of a) Mach number, b) gauge pressure, and c) vorticity under steady-state conditions.

injection-driven flow. For example, the Mach number increases linearly along the core from zero at the head end to 0.044 at the exit. The linear increase is consistent with cold-flow predictions for a similar chamber simulating a slab rocket motor.<sup>23</sup> According to cold-flow analysis, the mean velocity could be calculated from

$$u(x, y) = v_b \sqrt{F^2 + (x/H)^2 F'^2}$$
(3)

where  $v_b$  is the blowing or ejection velocity of the gases into the chamber. The large injection mean flow function is given by<sup>23</sup>

$$F = \cos\theta + (\pi/4)\varepsilon \left\{ \left( \phi \cos\phi - \sin\phi \right) \ln \left| \tan\left( \phi/2 \right) \right| + \cos\phi \left[ A\phi - I(\phi) \right] \right\} + O(\varepsilon^2)$$
(4)

$$I(x) = x + 2\sum_{k=1}^{\infty} \pi^{-2k} \left(1 - 2^{1-2k}\right) \frac{x^{2k+1}}{2k+1} \sum_{j=1}^{\infty} j^{-2k}$$
$$= x + \frac{1}{18} x^3 + \frac{7}{1800} x^5 + \dots$$
(5)

and the derivative is given by  $F' = \frac{1}{2}\pi \sin\theta + O(\varepsilon)$ . The asymptotic truncation order in Eq. (4) is gauged to the reciprocal of the injection Reynolds number,  $\varepsilon = \operatorname{Re}^{-1} = \nu/(\nu_b H) = O(10^{-3})$ . Here, the trigonometric arguments  $\theta = \frac{1}{2}\pi y/H$  and  $\phi = \frac{\pi}{2} - \theta$  are used for brevity. Based on this laminar mean flow solution, the Mach number can be determined from

$$M(x, y) = M_b \sqrt{\cos^2 \theta + \frac{1}{4} x^2 \pi^2 \sin^2 \theta} + O(\varepsilon M_b)$$
 (6)

which, along the chamber midsection, becomes  $M(x,H) = \frac{1}{2}\pi(x/H)M_{h} + O(\varepsilon M_{h})$ . Assuming that the bulk flow motion outside the flame zone can be approximated by the flow behavior of a non-reacting mixture (i.e., the propane-air gas products), one may set  $v_{h} = v_{f}$  in the cold-flow equations. Thus, based on Eq. (6), the local Mach number can be expected to vary linearly along the midsection plane from zero to  $\frac{1}{2}\pi(1/0.05)1.36 \times 10^{-3} = 0.043$  at aft end. This result is clearly consistent with the downstream value displayed in Fig. 2a (although not shown on the graph, the numerical simulation predicts a maximum value of 0.0439). Had the injection speed enhancement due to chemical reactions been ignored, the Mach number prediction at the downstream end would have been significantly underestimated (in this case, to 0.018). We thus conclude that, despite the adequacy of cold-flow models in approximating the gas motion outside the flame zone, the thermally adjusted speed  $v_f$  should be used to represent the equivalent blowing speed rather than the wall injection speed  $v_w$ . Otherwise, the usefulness of a cold-flow model can be severely compromised. The increase in effective gas injection can be attributed to the presence of chemical reactions and accompanying volumetric dilatations that are not present in the equivalent cold-flow model.

In like fashion, the pressure contours in Fig. 2b display an almost one-dimensional distribution that is weakly sensitive to the downstream location. However, no discernible variation in the transverse direction is observed, except for the small change arising from the volume dilatation in the near-wall flame zone. These observations may be attributed to the low injection Mach number and can be corroborated by the theoretical results obtained without chemical reactions. In fact, based on Majdalani and Van Moorhem,<sup>23</sup> the mean-pressure varies according to

$$p(x, y) = P - \frac{1}{2}\rho v_b^2 \left[\frac{1}{4}\pi^2 (x/H)^2 (1 + \frac{1}{2}\pi\varepsilon\sin\theta) + \cos^2\theta\right]$$
(7)

Along the midsection plane, it is clear that the transverse variation is manifested by the  $\cos^2 \theta$  term, which is too small to be considered. On the other hand, the diminution in gauge pressure along the chamber length can be calculated to be  $\frac{1}{8}\pi^2 \rho v_b^2 (x/H)^2$ . Unlike the cold-flow simulation where all thermostatic properties are kept constant, the density in the actual simulation varies along the chamber axis. At x = L, a density of 0.182 kg m<sup>-3</sup> can be used to predict a pressure drop of nearly 120 Pa. This value concurs with the actual pressure decrease recorded in the actual simulation and shown in Fig. 2b (the maximum recorded pressure was 125 Pa).

#### B. Mean Flow Vorticity

To understand the sources of mean flow vorticity, attention can be turned to the Crocco-Vazsonyi equation. The material derivative of vorticity is expanded into

$$\frac{\mathbf{D}\boldsymbol{\Omega}}{\mathbf{D}t} = (\boldsymbol{\Omega}\cdot\nabla)\boldsymbol{u} - \boldsymbol{\Omega}(\nabla\cdot\boldsymbol{u}) - \frac{(\gamma+1)}{(\gamma-1)}\nabla V \times \nabla p + v\nabla^{2}\boldsymbol{\Omega} - \mu\nabla V \times (\nabla\times\boldsymbol{\Omega}) + \frac{4}{3}\mu\nabla V \times \nabla(\nabla\cdot\boldsymbol{u})$$
(8)

where  $\boldsymbol{\Omega} = \nabla \times \boldsymbol{u}$  and  $V = 1/\rho$  is the specific volume. In the present two-dimensional simulation, the vortex stretching mechanism  $(\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{u}$  disappears. Vorticity is generated at the wall because of the no-slip condition that forces the parallel component of velocity to vanish along the porous surface.<sup>18</sup> Vorticity varies rapidly in the flame zone where both the volume dilatation  $\boldsymbol{\Omega}(\nabla \cdot \boldsymbol{u})$  and baroclinicity  $\nabla V \times \nabla P$  become important. Their influence is evidenced in Fig. 2c by the sudden emergence of vorticity overshoot near the surface.

The last two terms in Eq. (8) represent viscous damping and both appear to be secondary because of the large injection Reynolds number and the small mean flow velocity gradients in the entire chamber. In the absence of viscosity, baroclinicity, and volumetric dilation outside the flame zone, the mean flow vorticity shown in Fig. 2c appears to be nearly conserved while drifting downstream. Its magnitude increases almost linearly from zero at the head end to  $1152 \,\text{s}^{-1}$  at the aft end. To a good approximation, it can be evaluated from<sup>23</sup>

$$\left|\Omega\right| = \frac{1}{4}\pi^{2}(x/H^{2})v_{b}\cos\theta + O(\varepsilon) \approx 1000v_{b} = 1154\,\mathrm{s}^{-1}$$
(9)

This value concurs with the mean vorticity at the aft end. Here too, the use of a non-reactive formulation for the bulk vorticity outside the flame zone seems appropriate.

#### C. Mean Temperature and Heat Generation

To complete our coverage of flowfield attributes, the mean temperature distributions are presented in Fig. 3 at various axial locations ranging from x/H = 2 to 17. The proximity to the head end is purposefully targeted to ensure laminar conditions.<sup>12</sup> Due to the uniformly distributed heat of reaction along the surface, the overlaid temperature curves appear to be independent of the axial location. Rather, a one-dimensional variation in the y-direction is observed. The weak thermal sensitivity to the axial position can be attributed to the geometric aspect ratio and larger gradients in the transverse direction. Apparently, the downstream flow convection exerts hardly any influence on the flame structure. As shown in the graph, the temperature increases from the wall injection value of 350 K to a maximum of 1938 K just past the flame edge, at  $y = \delta_f = 2 \,\mathrm{mm}$ . The thickness  $\delta_n$  is used to denote the premixed flame standoff distance, which is approximately 0.9 mm. Due to the rapid inward convection, this maximum temperature spreads uniformly across the core. The climbing temperature segment shown in Fig. 3 is sustained by a non-zero heat input curve for  $\dot{Q}$ .

It may be useful to note that the calculated flame temperature of 1938 K agrees with the chemical equilibrium result of 1915 K predicted by the NASA CET93 code.<sup>14</sup> The slight discrepancy may be due to



Fig. 3 Mean temperature and heat release above the porous surface at several stations corresponding to: x/H = ----2, ------7, -----12, ------17.

the more detailed reactions incorporated in the CET93 calculations. The injection velocity of 0.206  $ms^{-1}$  agrees well with the experimental value of 0.209  $ms^{-1}$  measured by Metghalchi and Keck.<sup>24</sup> This agreement further supports the current methodology based on global chemical kinetics.

The typical S-shaped temperature curve displayed in Fig. 3 can be ascribed to the expected thermal surge across the flame zone. The retention of an almost-constant core temperature is caused by the strong convective action that brings the hot products closer to the core. As demonstrated by Majdalani and Van Moorhem,<sup>23</sup> the transverse penetration of injectants can be significant, especially in planar configurations, where the lack of curvature enables the injected mixture to draw nearer to the core before merging into the axial stream.

#### **D.** Simple Asymptotic Solution

Using standard descriptors, the differential forms of mass, momentum, and energy conservation can be expressed by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \tag{10}$$

$$\rho \left[ \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla) \boldsymbol{u} \right] = -\nabla p + \mu \nabla^2 \boldsymbol{u}$$
(11)

$$\rho C_{p} \left[ \frac{\partial T}{\partial t} + (\boldsymbol{u} \cdot \nabla)T \right] - \left[ \frac{\partial p}{\partial t} + (\boldsymbol{u} \cdot \nabla)T \right] = k \nabla^{2}T + \dot{Q} \quad (12)$$

where  $\rho$ , p, u = (u,v), T, and t represent the mean flow density, pressure, velocity, temperature, and time, respectively. While the heat source  $\dot{Q}$  is used to represent the net rate of exothermic chemical reactions, the remaining properties represent the specific heat  $C_p$ , viscosity  $\mu$ , and thermal conductivity k.

Equations (10)–(12) can be simplified based on the assumptions that:

(1) Gradients in temperature normal to the porous walls are larger than axial gradients  $(\partial T / \partial y \gg \partial T / \partial x)$ . This assumption is consistent with the multidimensional simulation results.

(2) Thermal diffusion is small when compared to thermal convection and heat generation. Such a relative comparison can be accomplished using an order of magnitude analysis.

(3) The velocity inside the flame zone has a negligible axial component because of the no-slip condition at the wall where the parallel x – component of u vanishes.

(4) The bulk velocity outside the flame zone is adequately represented by the isothermal mean flow solution based on the thermally-enhanced blowing speed  $v_f$ . This mean flow solution has been shown in<sup>25</sup> to provide a sufficiently accurate approximation.

Under these underlying conditions, Eqs. (10)–(12) reduce to

$$\frac{\partial(\rho v)}{\partial y} = 0 \text{ or } \rho v = \text{constant}$$
 (13)

$$\rho v C_p \frac{\partial T}{\partial y} = \dot{Q} \tag{14}$$

The decoupling of the momentum equation can be attributed to the one-dimensionality of the velocity distribution inside the flame zone. Equations (13)–(14) exhibit the following arrangement of boundary conditions:

at y = 0 (imposed),  $\rho = \rho_w$ ;  $v = v_w$ ;  $T = T_w$  (15)

at 
$$y = \delta_f$$
 (deduced),  $\rho = \rho_f$ ;  $v = v_f$ ;  $T = T_f$  (16)

While conservation of mass dictates that  $\rho v = \rho_w v_w$ inside the thin flame zone, conservation of energy cannot be used without providing information regarding the manner in which chemical energy is spatially released over the interval  $0 \le y \le \delta_f$ . The analytical model must be based on the prescribed form of heat released by chemical reactions. This form needs to be determined beforehand, either experimentally, or by simulation. In the numerical model, the exothermic heat of reaction associated with propane-air combustion is considered. This leads to a typical skewed symmetric function that exhibits a peak value inside the flame zone. Similar heat of reaction curves have been associated with other combusting hydrocarbons whose resolution has invariably demanded numerical integrations.26-29

The heat release curve obtained by the numerical simulation can be accurately represented by a Gaussian distribution. Gaussian distributions and error functions have been employed previously to describe both temperature and concentration profiles in several documented studies. Using single reactions and identical species diffusion coefficients, Borghi and Destriau<sup>30</sup> have defined their sum of mass fractions via Shvab-Zeldovich functions that can be represented by Gaussian distributions. Warnatz, Maas, and Dibble<sup>31</sup> have also shown that, for a species transport equation in which convective action is equally matched by diffusion, the solution may be expressed in terms of error functions. For near equi-diffusion flames (NEF) that arise in the vicinity of stagnation points, Buckmaster and Ludford<sup>32</sup> have developed an expression for the temperature profile using error functions as well. Their solution has been based on the coupled equations representing a balance between species diffusion, thermal diffusion, and convective transport of species and heat. In the current study, the heat source is represented by the Gaussian form

$$\dot{Q}(y) \equiv C_1 + C_2 \exp[-C_3(y - C_4)^2]$$
 (17)

where the characteristic coefficients  $C_i$ , i = 1,...,4 can be determined from the heat of formation of diverse species involved in the combustion of a given fueloxidizer mixture.

Equation (17) can be substituted into Eq. (14) and integrated. Using Eq. (15) to specify the single temperature boundary condition at the wall, one obtains

$$T(y) = T_w + \frac{2C_1\sqrt{C_3}y + C_2\sqrt{\pi}}{2\sqrt{C_3}\rho v \overline{C}_p} \times \left\{ \operatorname{erf}\left(C_4\sqrt{C_3}\right) + \operatorname{erf}\left[(y - C_4)\sqrt{C_3}\right] \right\}$$
(18)

In order to allow for comparisons between Eq. (18) and computations that take into account global chemical kinetics, the numerically generated heat release function is used as an example. Using the method of least-squares, the corresponding coefficients in Eq. (17) are found to be

$$C_1 = 6.34 \times 10^6; C_2 = 1.061 \times 10^9;$$
  

$$C_3 = 22.0 \times 10^6; C_4 = 0.00093251.$$
(19)

In order to evaluate the temperature distribution, the density and injection velocities at the wall are taken as  $\rho_w = 1.02 \text{ kgm}^{-3}$  and  $v_w = 0.2 \text{ ms}^{-1}$ .<sup>13</sup> The specific heat used in Eq. (18) is approximated by the average specific heat of the reactants. This is due to the flame zone being predominantly occupied by the reactants. The gas products, it has been shown, occupy the majority of the domain outside the flame region. The reactant mixture  $C_p(T)$  is determined using the sum of species specific heats weighed by individual mass fractions. For propane-air reactants, we use

 $C_p(T) = \sum Y_i C_{p_i} = Y_{C_3H_8} C_{pC_3H_8} + Y_{O_2} C_{pO_2} + Y_{N_2} C_{pN_2}$  (20) where  $Y_i$  represents the mass fraction of species *i*. The average specific heat of the reactants is then calculated from

$$\bar{C}_{p} = \frac{1}{T_{f} - T_{w}} \int_{T_{w}}^{T_{f}} C_{p} \, \mathrm{d}T$$
(21)

where  $T_w$  and  $T_f$  are 350 K and 1938 K, respectively.<sup>13</sup>

Having determined the means to approximate the spatial evolution of the temperature, attention is turned to the thermally-enhanced gas velocity directly above the flame edge. Using mass conservation and the ideal gas law under isobaric conditions, one finds

$$v_f = \frac{\rho_w}{\rho_f} v_w = \frac{T_f}{T_w} v_w \tag{22}$$

Consequently, given the injection temperature and velocity at the wall, the key for determining the thermally-enhanced velocity lies in the accurate prediction of the flame temperature  $T_f$ .

Fig. 4a compares the analytical approximation for Q(y), given by Eq. (17), and the numerical prediction.<sup>13,25</sup> Also shown in Fig. 4b are the temperature distributions obtained analytically, from Eq. (18), and numerically, based on Chu.<sup>13</sup> While the analytical solution matches the simulated temperature at the wall, it closely approximates  $T_f$  at the flame edge (i.e., at  $y = \delta_f$ ). Outside the flame zone, the numerical solution yields a constant temperature of 1938K, while Eq. (18) predicts 1935K. Coincidentally, the analytical value is closer to the flame temperature of 1915K procured from NASA CET93;14 the latter utilizes more detailed chemical reactions than those employed by Chu.<sup>13,25</sup> Note that, since the analytically predicted  $T_f$  is close to the assumed value used in Eq. (21), there is no need to re-evaluate the average specific heat. In the absence of a priori knowledge of  $T_f$ , however, the following algorithm may be used: (0) assume  $T_f$ ; (1) calculate  $\overline{C}_p$  from Eq. (21); (2) calculate  $T_f$  from Eq. (18); and, (3) repeat steps 1–2 until convergence.

It should be noted that, outside the flame zone, the analytical solution predicts a slightly increasing temperature in the inward direction. This is contrary to the numerical simulation results which predict a constant temperature throughout the chamber core. Interestingly, the slight increase in temperature outside the flame zone is often exhibited in laminar premixed flame profiles for methane-air mixtures and other combustibles described by Turns (cf. p. 272).<sup>26</sup> At the outset, it appears that the simple analytical model presented here can faithfully reproduce the fundamental thermal trends confirmed in previous studies.<sup>26-29</sup> Further refinements are necessary to reduce its limitations and broaden its applicability.

In like fashion, the validity of Eq. (22) can be verified for the specific problem of propane-air combustion. Whereas the velocity at the flame edge is calculated to be  $v_f = 1.15 \text{ ms}^{-1}$ , the approximate analysis based on Eq. (22) yields  $1.14 \text{ ms}^{-1}$ . The 1% error in the analytical prediction may be deemed reasonable, considering its relative simplicity in comparison to the finer flame zone detail captured by computational measurements.

#### V. Unsteady Flow Results

After steady-state solutions are reached, periodic flow oscillations are imposed at the head end in an effort to trigger acoustic waves at user-prescribed frequencies. The boundary conditions along the burning surface are left unchanged with respect to those used during steady-state simulations. Following other experimental and numerical benchmark studies,<sup>8-10</sup> the impressed oscillation frequency is varied from 100 to

800 Hz, while the amplitude of pressure fluctuations (*A*) is set at two percent of the mean pressure ( $\approx$ 160 dB). It should be noted that, according to acoustic theory,<sup>33</sup> the natural frequencies of the simulated chamber can be estimated from

$$f_m = ma_f / (2L); \ a_f = \sqrt{\gamma RT_f} = 846 \text{ m/s}$$
 (23)

where  $\gamma = 1.265$ ,  $R = 292 \text{ J kg}^{-1}\text{K}^{-1}$ , and  $T_f = 1938 \text{ K}$ . Note that, due to the small thickness of the flame zone, the vast portion of the chamber cavity is filled with product gases such as CO<sub>2</sub>, H<sub>2</sub>O, and air. For this reason, physical properties to be used in Eq. (23) are derived from the C<sub>3</sub>H<sub>8</sub>-air products whose molecular weight averages 28.47 kg kmol<sup>-1</sup>. Hence, for a unit chamber length, the problem's natural frequencies can be estimated from  $f_m = 423m$ , for m = 1, 2, ..., etc.

#### A. Unsteady Pressure and Velocity Fluctuations

Figure 5 illustrates the instantaneous isobaric contours of the fluctuating pressure and iso-velocity fields for f = 400 Hz. This value is only 5.4% smaller than the first fundamental frequency of the combustion chamber at m = 1. Here the fluctuating variables are obtained by subtracting mean values, calculated over a complete cycle of oscillation, from their total time-



Fig. 4. Spatial distribution of a) the chemical rate of heat release  $\dot{Q}(y)$ , and b) the steady-state temperature T(y) based on computational fluid dynamics (- - -) and analytical solutions (- -). Note that for  $y > \delta_f$ , the reaction heat released is negligible.

dependent quantities. As a result of this sifting operation, a well-defined traveling wave motion is realized. The wave motion bears a striking resemblance to the standing mode shape described by Majdalani.<sup>34</sup> Therein, the physical setting is identical except that a non-reacting gas is used. Nonetheless, the analytical predictions obtained under laminar conditions seem to agree quite favorably with the current results.<sup>34</sup>

With respect to the unsteady pressure and velocity, Fig. 5 suggests the same overall mode shape by exhibiting a pressure node and a velocity anti-node midway along the chamber length at x = 0.5L. The near-wall combustion interactions do not seem to exert any marked influence on the general character of the acoustic velocity, so long as the flow remains dominated by longitudinal strain rates.

It should be noted that the current contours are quite similar to those described analytically, based on the closed-form expression

$$u'(x, y, t) = \frac{A}{a_f \rho} \left[ \sin(m\pi x/L) \sin(m\pi a_f t/L) -F \sin(m\pi xF/L) \exp(\zeta) \sin(m\pi a_f t/L + \Phi) \right]$$
(24)

where

A = 2000 Pa,  $a_f = 846 \text{ ms}^{-1}$ , m = 1,  $\omega = 2\pi f_m$ , (25) and

$$\zeta = -\xi \int_0^{y/H} F^{-3}(x) dx$$
$$= -\frac{1}{\pi} \xi \left\{ \ln \tan\left[\frac{1}{2}(\theta + \frac{\pi}{2})\right] + \sec \theta \tan \theta \right\}.$$
(26)

The viscous parameter is

$$\xi = v\omega^2 H v_f^{-3} \tag{27}$$



Fig. 5 Contour plots of instantaneous acoustic pressure and axial velocity fluctuations at 400 Hz.

$$\Phi = -\operatorname{Sr} \int_{0}^{y/H} F^{-1}(x) dx = -\frac{2}{\pi} \operatorname{Sr} \ln \tan \left[ \frac{1}{2} \left( \theta + \frac{\pi}{2} \right) \right]; \quad (28)$$

where the Strouhal number is, as usual, given by

$$Sr = \omega H / v_f \tag{29}$$

As seen in Fig. 5, rotational effects are lifted off the burning surface by approximately 10% due to the presence of flame interactions. In addition to the expected coupling effects between acoustic and vorticity waves near the surface, the velocity fluctuations seem to exhibit an additional twodimensional correction resulting from interactions between the vortico-acoustic wave and the non-uniform temperature distribution across the flame zone. Away from the surface, vortical and thermal effects quickly decay and are dominated by a simple inviscid wave structure. This is manifested in Fig. 5 by a onedimensional acoustic velocity distribution over the range  $0.4 \le y/H \le 1$ . The unsteady wave amplitude of approximately 13 ms<sup>-1</sup> can be interpreted in light of the established theory of aeroacoustics;<sup>18</sup> accordingly, an acoustic velocity amplitude of  $A/(a_f \rho) \approx 13 \text{ ms}^{-1}$  can be expected near the core. However, due to the thermal suppression of vorticity waves across the flame zone, a relatively small overshoot in unsteady velocity is observed near the wall. This overshoot is caused by the pairing of vorticity and acoustic waves and is much smaller than the typical 100% overshoot value reported in cold-flow simulations. The attenuation of the vorticity wave is due, in part, to the strong volume dilatation across the flame zone.

The formation of acoustically induced shear layers are examined in Fig. 6 by plotting the transverse distributions of both amplitudes and phases of the fluctuations u', v', and T' at different frequencies. The flame location is also included to facilitate the discussion. The most salient features are the shortening in spatial wavelength ( $\lambda$ ) and the reduction in wave amplitude with frequency. These observations are in agreement with former studies of non-reacting flows.<sup>23,34-37</sup> For example, based on theoretical grounds, the diminution in wavelength may be inferred from Eq. (24) to be  $\lambda = v_f F / f_m$ . This relation clearly displays an inverse relation between  $\lambda$  and  $f_m$ . The reduction in total wave amplitude with frequency is attributable to the reduced wavelength which, in turn, causes the coupling between acoustical and vortical waves to take place nearer to the flame zone. There the phase interference with thermal waves leads to a faster attenuation of vortical strength. Outside the flame zone, the vortical presence is further exacerbated by the increased shearing, reversal, and damping of fluid particles over shorter length and time scales.

The observed behavior may once more be illuminated by the Crocco-Vazsonyi equation. Using similar arguments to those given in Sec. IV to explain the mean flow vortical growth, the interplay between volume dilatation and baroclinicity in the flame zone can be used to explain the attenuation of unsteady vortical growth. In addition, the wavelength of the vortical motion is increased in the flame zone due to the increasing wave propagation speed (partly triggered by volumetric dilatation). At the outset, the combination of volume dilatation, baroclinicity, and increased wave propagation speed in the flame zone tend to reduce the role of vorticity near the wall, especially as compared to the case of a non-reacting flow. This explains why the largest unsteady amplitudes occur outside the flame zone and later decay as the core is approached. In particular, note that, despite the presence of chemical reactions, the amplitude of v' is smaller than that of u'by the order of the Mach number,  $M_f = O(10^{-3})$ . This result is significant as it lends support to the theoretical predictions obtained from cold-flow studies of the core flow.<sup>34</sup>

#### **B.** Velocity Decomposition

In order to better understand the coupling between the flame response and velocity waves, it is helpful to identify the separate contributions of pressure-driven, boundary-driven, and entropy-driven disturbances. Chu and Kovásznay's<sup>38</sup> mode decomposition approach can be implemented to subdivide the total velocity fluctuation into three parts representing acoustical, vortical, and thermal oscillation modes. This approach has been previously employed by Flandro<sup>18</sup> and Majdalani,<sup>34</sup> who have separated the velocity field into two parts: an irrotational, acoustical  $u'_a$ , and a rotational, vortical,  $u'_{v}$ . Since the current study involves chemical reactions, the contribution from heat fluctuations must be added in the form of a thermal, entropy-driven correction  $u'_s$ . The total fluctuation can hence be split into

$$u' = u'_a + u'_v + u'_s \tag{30}$$

On the one hand, the acoustic velocity  $u'_a$  is directly correlated with the pressure fluctuation through the acoustic impedance,  $\overline{\rho a}$ . This product varies rapidly in



Fig. 6 Amplitudes and phases of unsteady velocity and temperature fluctuations at x/H = 3 and four frequencies corresponding to: f = ----100 Hz, ------200 Hz, ------800 Hz.

the flame zone and is proportional to the inverse of  $\sqrt{T}$ . On the other hand, the rotational part  $u'_{\nu}$  incorporates the effects of unsteady vorticity generation and transport. The newly introduced velocity fluctuation  $u'_s$  results from the unsteady heat release originating in the flame zone. Based on acoustic theory,<sup>33</sup> this quantity can be approximated by

$$u'_{s} \approx \frac{k}{\overline{\rho}C_{p}\overline{T}} \frac{\partial T'_{s}}{\partial x}$$
(31)

where  $T'_s$  represents the non-isentropic temperature fluctuation. Since the temperature gradient only exists in the transverse direction, the effect of entropy waves on the axial velocity fluctuation can be ignored. Its influence on the transverse velocity fluctuation must be retained, however, so long as it is capable of exciting large transverse oscillations.<sup>13</sup>

#### C. Unsteady Temperature Fluctuations

Figures 6e-f illustrate the effect of frequency on the amplitude and phase of temperature fluctuations. The corresponding steady-state distributions of temperature and heat release rate have already been examined in Fig. 3. Pursuant to the velocity decomposition, the temperature fluctuation can be obtained by superimposing the acoustic and entropy-driven contributions. The acoustic component  $T'_a$  obeys the isentropic relation governing the acoustic pressure. In the current study, it has an amplitude of 8 K in the core region. The non-isentropic component  $T'_s$ is approximately proportional to the product of entropy fluctuation and mean temperature.<sup>33</sup> In view of Eq. (31). it can be expressed by

$$T'_{s} \approx \dot{q}' \tau_{c} / C_{p} \tag{32}$$

where  $\dot{q}'$  is the fluctuation in the rate of chemical heat release, and  $\tau_c$  represents its associated characteristic time. As shown in Fig. 6c, the fluctuating temperature response exhibits a peak directly below the line of maximum mean temperature. This peak reaches about 60 K at a frequency of 100 Hz and then drops to nearly 35 K at 800 Hz. The thermal oscillations also persist longer and therefore, penetrate deeper into the chamber core at lower oscillation frequencies. This result can be attributed to the intimate coupling near the wall with the longitudinal vortico-acoustic waves whose role is to sustain these thermal oscillations. Since the vorticoacoustic depth of penetration diminishes at higher frequencies, so will the depth of thermal fluctuations. This effect is observed in both modulus and phase variations in Fig. 6c.

So long as the vortico-acoustic waves cause the flame to flutter in the vicinity of the wall, thermal fluctuations can be sustained. These fluctuations are empowered by alternating high-temperature sweeps into the preheat zone that can periodically expedite the ongoing chemical combustion and attendant heat of reaction. The character of thermal fluctuations near the wall can be adequately represented by the Arrhenius relation connecting reaction rates and temperatures. When sufficiently removed from the flame zone (where heat is constantly supplied), the unsteady temperature becomes dominated by the high-capacity, pressure-driven 'masslike' response  $T'_a$  that always prevails in the core region.38 Between the flame edge and the core, the entropy fluctuation  $T'_s$  is smoothed out due to heat conduction and the thermal flywheel effect that accompanies the traveling acoustic wave.

Since the flame oscillation is induced by the vorticoacoustic motion, the characteristic time  $\tau_c$  described in Eq. (32) may be estimated by the period of acoustic oscillations,  $\tau_c \approx 1/f_m = 2L/(ma_f)$ . Hence, a lowfrequency acoustic motion will lead to a largeramplitude temperature fluctuation while allowing a deeper penetration of the vortical wave into the chamber core. These theoretical predictions are clearly reflected in the current numerical results.

#### D. Response to Unsteady Heat Addition

The unsteady heat release rate is shown in both amplitude and phase in Fig. 7 at one axial position and four different frequencies. For the reasons given to explain the behavior of the temperature fluctuations, the amplitude and depth of penetration of the heat waves decay more rapidly at higher frequencies. However, two uneven peaks in Q are observed below and above the flame location. The value of the peak outside the flame zone is approximately 60% of the inner one, which is consistent with the experimental observation of Sankar et al.<sup>2</sup> From a theoretical perspective, the two energy peaks serve as a combination of an acoustic monopole and a dipole source for driving the observed oscillations.<sup>33</sup> In order to explain the observed damping outside the flame zone, attention can be turned to the fundamental role of heat addition in driving or suppressing the unsteady fluctuations.

In principle, the amplification of an acoustic wave p' due to periodic heat input supplied at a rate  $\dot{q}'$  can be characterized using Rayleigh's parameter,<sup>39</sup>

$$\langle p'\dot{q}' \rangle = \int_{\tau_c} p'(\mathbf{x},t) \dot{q}'(\mathbf{x},t) dt$$
 (33)

An impressed acoustic excitation will grow when the above parameter is positive. Thus, if heat is added at the moment when the combustion mixture is locally expanding (i.e., during a favorable pressure phase), the dilatational thermal energy is positively gained insofar as it leads to a more pronounced volumetric expansion. When this occurs, the thermal wave can be considered to be in favorable phase with the acoustic motion that it tends to sustain and amplify. Conversely, if heat is added at the moment of local gas compression (i.e., during an adverse pressure phase), the effect of thermal addition is that of opposing the volumetric contraction. Wave cancellation takes place as some of the acoustic energy is eliminated.

In the problem at hand, two regions of positive and negative Rayleigh numbers can be recognized. As shown in Fig. 8, the two regions of favorable and adverse thermo-acoustic coupling are delineated by the flame standoff distance. These regions are identified by plotting the Rayleigh parameter  $\langle p'\dot{q}' \rangle$  at x/H = 3and the same four frequencies. In both regions, the magnitude of  $\langle p'\dot{q}' \rangle$  diminishes at higher m, indicating a weaker thermo-acoustic coupling at higher frequencies. The weaker coupling at higher frequencies may explain the diminution in temperature oscillations at higher  $f_m$  in Fig. 6c. In particular, the negative Rayleigh number for  $y > \delta_p$  is indicative of a damped out region in which the role of unsteady heat addition switches to that of vortico-acoustical wave attenuation. This feature may explain the damping character of unsteady energy release in the outer flame region (y > z0.9 mm).

#### VI. Conclusions

This study has focused on the interactions between acoustic waves and premixed flame dynamics in porous chambers. In the process, the complete conservation equations were numerically simulated while accommodating variations in thermo-physical properties. Finite-rate chemical kinetics are also enabled to spatially resolve the flame structure. Both mean and unsteady flame attributes are examined in various oscillatory flow environments. The unsteady motions are found to comprise contributions from acoustical, vortical, and thermal modes that strongly depend on the frequency of the impressed acoustic wave. Out of the three disturbance types, thermal oscillations are restricted to a thin region near the wall. Outside this flame zone, the spatial distribution of pressure and velocity closely resemble the theoretical predictions obtained from cold-flow solutions based on the flame-enhanced gas injection speed. In particular, the flow is found to be mainly isothermal and the



Fig. 7 Amplitudes and phases of unsteady chemical heat release rate at x/H=3 and the same four frequencies shown in Fig. 6.



Fig. 8 Normalized Rayleigh parameter at x/H = 3and the four frequencies of Fig. 6.

fluctuating velocity in the transverse direction appears to be smaller than the axial component by the order of the Mach number  $M_f$ . Furthermore, the amplitude of all fluctuating components decreases at higher frequencies. This behavior applies to the acoustic boundary layer thickness as well. To explain the pairing between thermo-acoustic and vortico-acoustic waves, an energy cascade among these three separate fluctuation modes is identified. Within the flame standoff distance, the thermo-acoustic response is found to be favorable to vortico-acoustic growth. Outside this region, however, a negative Rayleigh number is confirmed, signaling an adverse thermo-acoustic coupling. As a result of the thermal oscillations induced by the vortico-acoustic wave, the rate of unsteady heat release is found to exhibit two uneven peaks. These can be attributed to the Arrhenius type of temperature dependence on reaction rates. The two peaks can also be viewed as a combination of an acoustic monopole and a dipole source for driving flow oscillations. Finally, the amplification and suppression roles of thermo-acoustic coupling due to unsteady heat addition are realized by calculating local Rayleigh numbers.

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