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In this article, the modeling of heat sinks is carried out using electronic cooling software under both forced and natural convection conditions. Simulations are applied to both detailed and compact heat sink models. The compact models are based on the volume resistance approach also known as the porous block model. These models are devised with the intention of approximating actual heat sink behavior while providing substantial savings in computational effort. In some cases, results obtained from numerical simulations are compared with independent laboratory measurements. This is accomplished with both forced and natural convection flow conditions. In the case of forced convection, the effective thermal conductivity of the compact heat sink is calculated directly from the Nusselt number correlation for flow over a flat plate. When this value is used, the numerically determined thermal resistance of the compact heat sink is found to compare well with the experimental value. In the case of natural convection, the effective thermal conductivity of the compact heat sink is calculated using the corresponding Nusselt number correlation for free convection over a vertical plate. The increased complexity of the correlation for free convection necessitates an iterative solution for the effective thermal conductivity. In this case also, it is shown that results obtained from CFD simulations compare favorably with available laboratory measurements. The case-studies illustrate the manner in which compact heat sinks can be conveniently used to reduce the needed computational cost, time and resources while producing sufficiently reliable results.

I. Introduction

HEAT sinks play an important role in cooling printed circuit boards (PCBs) and other power sources within computers and electronic enclosures. Since the turn of the century, industrial design has witnessed a proliferation in the use of heat sinks that require detailed modeling via Computational Fluid Dynamics (CFD). In cases involving large populations of high power components, the use of finite discretization in evaluating heat sink performance has proven impractical in view of its excessive demand for CPU and memory-allocation resources. Despite modern leaps in computer technology, the difficulty in modeling increasingly smaller and more complicated arrays of microchips has constantly caught up with processing speeds of computational tools. Due to the practical challenges in simulating large assemblies of coupled heat sinks, compact models have been introduced in recent years for the purpose of facilitating the analysis of large multichip modules (MCMs) used in high-power application specific integrated circuits (ASICs).

The idea of a compact heat sink is analogous to the lumped mass concept used in transient heat conduction studies. The notion is to replace the finned heat exchanger with a simpler element that can be more easily modeled as a single thermal resistor. In principle, this substitution may be viable if the simpler element is capable of providing the same thermal and fluid resistance properties as those of the actual device. So far, several equivalent models of varying degrees of accuracy have been proposed and used successfully by Krueger and Bar-Cohen,[1] Culham, Yovanovich and Lee,[2,3] Linton and Agonafer,[4] Patel and Belady,[5,6] Narasimhan and Kusha,[7] Narasimhan and Majdalani,[8,9] and others. In general these models rely on either the flat plate boundary layer or the porous block approach.

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As described in detail by Culham, Yovanovich and Lee,[2,3] the flat plate idea may be applied in one of four distinct ways in which the equivalent surface area of the simpler model is that of a) the base plate, b) the extended base plate, c) the raised fin, or d) the raised fin with base plate assembly. For a given heat dissipation rate $\dot{Q}$ and temperature excess ΔT between the base and the cooling fluid, $R_T = \Delta T / \dot{Q}$ may be used to represent the overall thermal resistance of the actual heat sink. According to the type-a model, an equivalent convection heat transfer coefficient h_e is determined such that $h_e A = 1/R_T$, where A is the area of the base. When this artificially increased heat transfer coefficient is assigned to the base plate, it leads to the same thermal resistance and, therefore, temperature drop, obtained in the actual finned assembly. According to the type-b model, an equivalent base area A_e can be calculated such that $hA_e = 1/R_T$, where h is the convection heat transfer coefficient from the base plate to the surrounding fluid. In this fashion, the reduced thermal resistance due to finning is implemented by increasing the surface area of the base plate without modifying the plate-to-coolant heat transfer coefficient. Similar ideas are employed in developing the type-c and type-d models.

Unlike the flat plate model that assigns the equivalent thermal resistance to a two dimensional surface, the porous block model assigns the thermal resistance associated with an actual heat sink to the three-dimensional volume of fluid that was once occupied by the fins above the base plate. As described by Patel and Belady,[5,6] reducing the thermal resistance of the volume of fluid above the base plate can be accomplished by increasing its thermal conductivity. An effective thermal conductivity k_e can thus be determined and distributed over the volume of fluid in a manner to reproduce the required thermal resistance R_T. Such an approach has been validated both numerically and experimentally by Narasimhan and Kusha [7], Narasimhan and Majdalani[8,9] and, more recently, by Narasimhan, Bar-Cohen, and Nair.[10,11] In the forced convection studies described by Patel and Belady,[5,6] Narasimhan and Kusha,[7] and Narasimhan, Bar-Cohen, and Nair,[11] an equivalent pressure loss coefficient needs to be additionally determined in order to emulate the flow resistance in the actual prototype. Conversely, in the free-convection case-studies presented by Narasimhan and Majdalani,[8,9] matching the smaller pressure loss incurred in the detailed heat sink becomes a lesser concern.

Due to the usefulness of compact models in reducing the computational effort in thermal optimization projects involving populated circuit boards, a growing interest in improving available methodologies can be witnessed today. This may be exemplified by the contemporaneous studies of Narasimhan, Bar-Cohen and Nair,[10,11] and those of Narasimhan and Majdalani.[8,9]

It should be noted that, in most of the studies cited above, standard heat transfer correlations are used in evaluating equivalent thermal properties. This is especially true of porous block models that require calculating the effective thermal conductivity k_e of the compact heat sink. The use of standard heat transfer correlations such as those given by Churchill and Chu[12] has become common-practice in most investigations concerned with the optimal design and selection of high performance heat sinks. Examples abound and one may consult with the works of Lee,[13] Knight *et al.*,[14,15] Wirtz, Chen and Zhou,[16] Mertol,[17] Le Jannou and Huon,[18] and others.

Surely the validity of using standard correlations in calculating the thermal properties of a compact heat sink remains an open ended question until sufficient comparisons between experimental and CFD simulations are made available. This paper consists of one such study aimed at illustrating the usefulness of a compact model in both forced and natural convection scenarios. In addition to one other paper by the authors[8,9] this article seems to present the second case-study in which the porous block model is applied to a buoyancy-driven air cooled heat sink. In this short article, the simulations will be conducted under laminar conditions only.

II. Description of the Models

Two strategies are used depending on whether the airflow inside the electronic enclosure arises naturally (i.e., from buoyancy currents) or whether it is 'pushed or pulled' by means of intake or exhaust fans. While the general modeling idea is the same in both cases, the corresponding empirical correlations differ. Due to the relatively simpler relations associated with forced convection, this type will be covered first.

A. Forced Convection Case-study

In the forced convection case, the heat sink assembly under consideration consists of 36 fins attached to a base of dimensions 76.2×76.2×3.175 mm^3 in a duct whose dimensions are 78.74×304.8×39.37 mm^3.[19] Along the solid boundaries of the duct, both no-slip and adiabatic conditions are imposed. The parallel-plate heat sink is made entirely of copper and is shown in detail in Fig. 1a.

While the fin height is 34.92 mm, the interfin gap is 1.575 mm. Heat generation is simulated by means of a steady planar source dissipating 100 watts. The source is placed at the center of the base under the heat sink. Across the inlet section to the heat sink, a uniform velocity of 1 m/s is imposed. This produces an interfin velocity of 2.1 m/s. Based on a calculated hydraulic diameter of 3.07 mm, a Reynolds number of 400 is obtained, indicating laminar conditions.

The forced convection simulations are also carried out using a compact heat sink model where the heat sink is replaced by an equivalent volume of fluid (i.e., a porous block) that can reproduce the same flow properties and heat transfer characteristics of the actual heat sink. The effective thermal conductivity k_e of the porous block is calculated from the Nusselt number correlation for forced convection over a flat plate. In order to calculate k_e, the following steps are taken.

First, the overall thermal resistance R_T of the compact heat sink is determined from the maximum surface temperature $T_{\max}$ of the actual heat sink. One uses

$$R_T = \Delta T / \dot{Q} = (T_{\max} - T_b) / \dot{Q} \tag{1}$$

where $T_b = (T_i + T_o)/2$ is the (bulk mean) coolant temperature based on the arithmetic average of inlet and outlet temperatures, T_i and T_o, respectively. Here

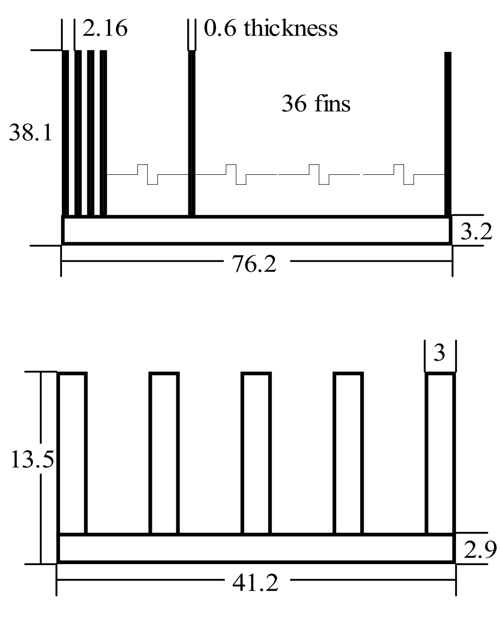


Fig. 1 Schematic of the extruded-fin heat sink used in a) forced and b) natural convection case-studies. The streamwise length in each case is 76.2 mm. All other dimensions are in mm.

$T_s = T_{\max}$ is the assumed surface temperature. Isothermity along the surface is justified by the large thermal conductivity of the base material, thus leading to insignificant temperature variations along the base plate. A similar idea is used in other works as well.[20-22] The justification for Eq. (1) is commented upon by Bar-Cohen, Elperin, and Eliasi.[23] It follows that, in the compact approach, the average temperature excess can be taken to be the maximum temperature difference between the base plate and the coolant. Evidently, two models that exhibit the same thermal resistance R_T may be expected to reproduce identical temperature drops. To find R_T, the maximum temperature at the base $T_{\max}$ must be numerically evaluated from the detailed heat sink simulation. This value is determined only once for a given heat dissipation rate generated by the source. The expediency of this technique is clearly illustrated in the detailed versus compact modeling flowcharts developed by Patel and Belady.[5,6] This approach becomes especially valuable when a validated numerical model for single chip or multiple chip packages (SCPs/MCPs) can be supplied by vendors. According to Bar-Cohen, Elperin, and Eliasi,[23] the availability of a reliable code for a given SCP can greatly facilitate its integration into the development of a competitive packaging design. This notion is further explored by Lasance, Vinke, and Rosten[21] who propose defining an idealized 'validation chip model' (VCM) that can reproduce the principal features associated with a given chip package.

Having determined the overall thermal resistance between the base plate and the coolant, the second step in our approach is to calculate the overall heat transfer coefficient $U = (R_T A)^{-1}$. Note that using an overall heat transfer coefficient instead of an effective convection heat transfer coefficient h_e constitutes a minor departure from the flat plate models described by Culham, Yovanovich, and Lee.[2,3] The symbol U is presently used because the thermal resistance R_T obtained from the detailed simulation incorporates both convective and radiative effects. This is due to both modes of heat transfer being accounted for in most commercial software packages used to determine $T_{\max}$ and, consequently, R_T and U. The interested reader may consult with Narasimhan and Majdalani[8,9] for more detail.

The third step in the volume resistance approach is to calculate the effective thermal conductivity of the compact heat sink from the Nusselt number correlation for forced convection flow over a vertical plate. Using standard descriptors and nomenclature, one may write[24]

$$\mathrm{Nu} = 0.664\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3} \tag{2}$$

where Nu, Re, and Pr are the classic Nusselt, Reynolds, and Prandtl numbers. Instead of using h_e in the expression for the Nusselt number, the overall heat transfer coefficient can now be substituted. Forthwith, an expression for the effective thermal conductivity of the compact heat sink is obtained. This is

$$k_e = 1.848\mu^{1/4}(UL)^{3/2}C_p^{-1/2}\left(\rho VL\right)^{-3/4}. \qquad (3)$$

In Eq. (3), V represents the average free stream velocity over a flat plate whose length is L in the streamwise direction. Hence, given the dynamic viscosity μ, specific heat C_p, and density ρ for the coolant at the film temperature $T_f = (T_{\max} + T_b)/2$, the effective thermal conductivity for the compact heat sink is at hand.

By assigning an artificially enhanced k_e to the block of fluid above the base plate, the thermal resistance of the actual heat sink is accurately reproduced by the compact model. In order to be complete, the compact model still has to emulate the flow resistance across the fin-delimited pathways. The fourth step is, therefore, to calculate the pressure loss coefficient that needs to be assigned to the compact model. This can be accomplished by curve fitting fundamental pressure loss correlations based on detailed heat sink simulations. By running the detailed heat sink simulations over a range of velocities, a correlation is developed relating the pressure drop across the actual heat sink to the air velocity, V. For a given heat sink configuration, a correlation is then developed, for each flow regime, by putting

$$\Delta p = \begin{cases} \frac{1}{2}\rho K_{\mathrm{lam}}V; & \text{laminar} \\ \frac{1}{2}\rho(K_{\mathrm{lam}} + K_{\mathrm{turb}}V)V; & \text{turbulent} \end{cases} \qquad (4)$$

where K_{lam} and K_{turb} represent the laminar or turbulent loss coefficients. From the curve-fitted loss coefficients, the linear or quadratic relations in Eq. (4) are then assigned to the compact model. Once the loss-coefficients are determined for a given heat sink, they need not be recalculated during the iterative design process, thus providing additional time savings.[5,6]

It should be noted that a similar quadratic relation has been recently developed by Narasimhan, Bar-Cohen, and Nair.[11] An equally useful approach is described, in a different application, by Teertstra, Yovanovich, and Culham.[25]

In principle, by sharing the same thermal conductivities and hydraulic loss coefficients, the detailed and compact heat sinks can be expected to yield equivalent results. In practice, a proper validation requires numerical and/or experimental verifications. The goal is for the compact model to project a chip temperature within $\pm 7\%$.[18]

B. Natural Convection Case-study

In the natural convection case-study, an AAVID heat sink (Model Number 62040) is considered. The heat sink is placed inside an enclosure with openings at both top and bottom ends. The enclosure is 53 mm high, 205 mm wide, and 381 mm long. Here also, simulations are first carried out using detailed heat sinks. For the model at hand, the actual heat sink consists of 5 aluminum fins and a base through which 8 watts of power are dissipated. The fin and plate dimensions are shown in Fig. 1b. In order to determine the properties of a porous block that can mimic the thermo-fluid response of the detailed heat sink, similar arguments to those presented in the forced convection case are offered. On that account, the effective thermal conductivity of the compact heat sink is calculated from the Nusselt number correlation for free convection flow over a vertical plate. Substituting the convection heat transfer coefficient by U, one may write, following Churchill and Chu,[26]

$$\mathrm{Nu}_L = \frac{UL}{k_e} = 0.68 + 0.67\mathrm{Ra}_L^{1/4}\left[1 + \left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{-4/9} \qquad (5)$$

Here Ra_L is the Rayleigh number given by

$$\mathrm{Ra}_L = g\beta L^3 \Delta T/(\nu\alpha) \qquad (6)$$

where g is the acceleration due to gravity, β is the coefficient of thermal expansion, L is the vertical length of the plate, ν is the kinematic viscosity, and α is the thermal diffusivity of the fluid. Due to the algebraic nature of Eq. (5), an exact expression for the effective thermal conductivity k_e of the compact heat sink does not seem possible. In fact, since k_e occurs on both sides of the equation, extracting a numerical value for k_e requires solving

$$\left(1 + ak_e^{9/16}\right)^{-4/9} k_e^{3/4} + bk_e - c = 0 \qquad (7)$$

where

$$a = \left[0.492/(\mu C_p)\right]^{9/16} b = 1.015/\lambda^{1/4}\ c = 1.49UL/\lambda^{1/4} \qquad (8)$$

and

$$\lambda = g\beta\rho^2 L^3 C_p \Delta T/\mu \qquad (9)$$

A short iterative routine is hence necessary to calculate k_e and thus complete the compact heat sink model in the natural convection case.

III. CFD Simulations

Pursuant to the foregoing model descriptions, numerical simulations are carried out for both forced and natural convection with either detailed or compact heat sinks. The CFD simulations are undertaken using a software package dedicated to the analysis of flow and heat transfer in electronic enclosures and chip cooling applications.[27] The steady-state flow and temperature fields in the computational domain are obtained by solving the three dimensional Navier-Stokes equations along with the energy equation based on a finite volume solver. The finite volume solver used by the code employs multigrid acceleration and relies on hexahedral elements. Multigrid acceleration reduces the error by iterating in an alternative fashion on a series of coarse and fine grids. The convergence criterion we have adopted is based on a tolerance that ensures that the residual error remains less than 10^{-3} in the momentum balance, and less than 10^{-7} in the energy balance.[28] Before launching the simulations, grid sensitivity tests are repeated until grid independence is realized.

IV. Results and Discussion

The robustness of the compact model is tested by comparing its results to experimentally acquired data along with detailed numerical simulations. A series of comparisons indicates that, for the compact model, an appreciable reduction in mesh size is gained at the expense of a small, tolerable loss in accuracy. Typical results obtained from different models are presented in Tables 1 and 2 below.

A. Overview

In Table 1, the thermal resistances obtained from the numerical simulations are compared with their corresponding experimental values. Using the experimentally measured value as a benchmark, the error in estimating the thermal resistance in the forced convection case increases from 8.8% to 9.3% when the compact model is used instead of the detailed system. Similarly, it increases from 5.5% to 6.5% in the natural convection case. It appears that a less than 1% reduction in accuracy arises when the compact model is used as a substitute. Incurring such an error can be a reasonable compromise for this and several other heat sink configurations. The main justification lies in the increased computational speed and reduction in grid sizing that accompanies the compact model. This result may be inferred from Table 2 where cell sizes used in both detailed and compact representations are posted for each of the forced and natural convection cases. Clearly, the order of magnitude reduction in the minimum number of cells needed for convergence seems to offset the small additional error entailed in the compact representation.

Table 1 Comparison of thermal resistance values obtained in experimental and CFD simulations

Simulation type	Forced mode R_T [KW^{-1}]	Natural mode R_T [KW^{-1}]
Experimental	0.43	6.50
Detailed CFD	0.46	6.86
Compact CFD	0.47	6.92

Table 2 Mesh size requirements

Case specification	Detailed model	Compact model	Gain factor
Forced convection	16898	1622	10.4
Natural convection	10499	4698	2.23

B. Forced Convection Model

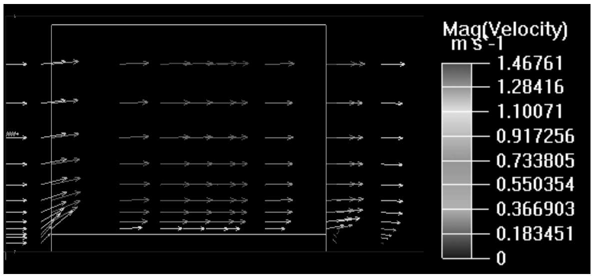
In the laminar forced convection case, CFD results are compared with the experimental data collected by Vogel.[19] Here, the effective thermal resistances obtained from the detailed and compact representations are found to be 0.46 and 0.47 $^{\circ}\mathrm{C/W}$, respectively. These values compare favorably with each other as well as with the experimental value of 0.43 $^{\circ}\mathrm{C/W}$ reported by Vogel.[19]

The agreement between theory and experiment in predicting overall heat transfer characteristics is reassuring but not sufficient. It is equally important to adequately simulate viscous and thermal boundary layer features. These features may be best illustrated by examining the velocity and temperature profiles in both detailed and compact heat sink models.

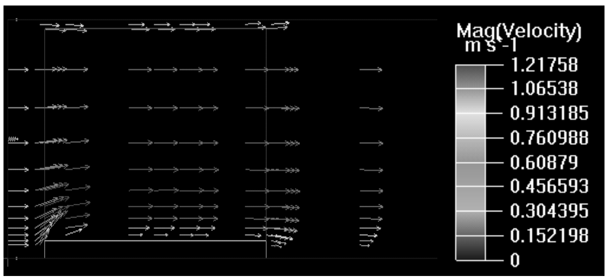
Before embarking on this endeavor, it may be useful to note that the temperature and velocity profiles at the outlet of the heat sink can differ depending on the model at hand. Matching thermal and fluid flow characteristics at the outlet section can be difficult. Nonetheless, this challenging process can be pivotal when modeling stacks of adjacent heat sinks. When such is the case, the outlet profile of one heat sink becomes the inlet profile of its downstream conjugate. In that event, accurate modeling of inlet and outlet conditions will require iterations and refinements that can affect the computational domain.

1. Velocity Profiles

With respect to flow properties, Figs. 2a–2b show the velocity profiles for the detailed and compact models, respectively. While the compact model underestimates the maximum velocity by 17%, both models lead to kinematically similar conditions. In fact, the magnitudes and orientations of the velocity vectors appear to be in a general kinematic agreement. This agreement may be attributed to the current ducted fin geometry that leads to a unidirectional flow channeling. For unducted heat sink arrangements, the avenue for bypass is more significant and the flow streamlines can be dissimilar. This is due to the current limitation of the porous block model in reproducing rectilinear flow streamlines.



a)



b)

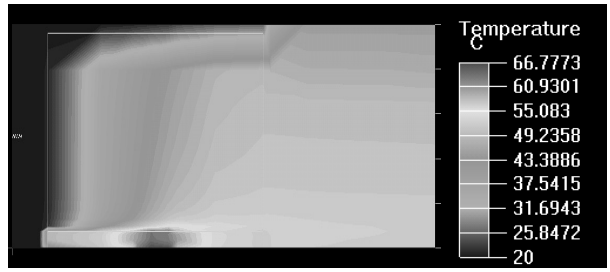
Fig. 2 Velocity profiles using a) detailed and b) compact heat sink models under forced convection.

2. Temperature Profiles

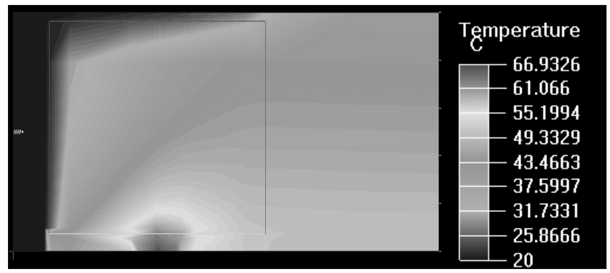
With respect to thermal properties, Figs. 3a–3b are used to illustrate the temperature profiles. It is reassuring to note the good agreement between both temperature distributions in which the compact representation overestimates the maximum temperature by 0.2%. This satisfactory agreement is the byproduct of accurate fluid flow and lumped thermal modeling. Our results are fortuitous as one might encounter difficulties when modeling arrays of heat sinks that exhibit numerous bypass avenues. In such instances, using pressure drop characteristics based on ducted wind tunnel correlations may be insufficient. Thus, despite its apparent attractions, the current approach has the disadvantage of exhibiting a velocity dependency that must be known prior to calculating the effective thermal conductivity of the compact heat sink. This step can be difficult to overcome in situations where heat sink arrangements arise in unducted wind tunnels. When this occurs, determining the proper velocity that leads to the correct effective conductivity can necessitate a challenging iterative process. More research is certainly required to determine better ways for modeling compact heat sink arrays exhibiting staggered arrangements.

C. Natural Convection Model

Numerical simulations are also carried out in the case of natural convection with both detailed and compact heat sinks. The Rayleigh number based on the



a)



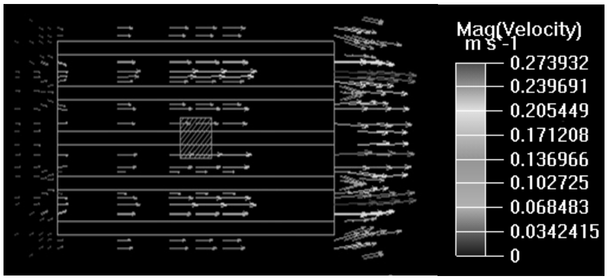
b)

Fig. 3 Temperature profiles using a) detailed and b) compact heat sink models under forced convection.

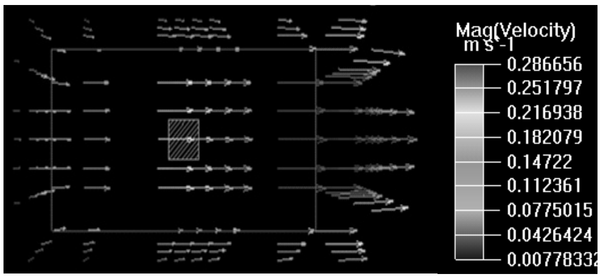
streamwise characteristic length of 76.2 mm is found to be 2×10^6, thus indicating laminar conditions. In the detailed representation, the heat transfer contribution due to radiation, which is now important, becomes a non-negligible factor. We find that the thermal resistance of 6.86 $^{\circ}$C/W obtained from the detailed analysis compares well with the experimental value of 6.5 $^{\circ}$C/W reported by Kusha, Rosenblat and Lee.[7] The discrepancy here is a mere 5.5%. Using the more expedient compact analysis, a thermal resistance of 6.92 $^{\circ}$C/W is calculated. This value falls within 6.5% of the actual measurement. When experimental uncertainty is factored in, it is clear that the 1% diminution in accuracy for using the compact model has a minor influence on the overall precision associated with the projected results.

1. Velocity and Temperature Profiles

Following the discussion in Sec. IV.B, we now turn our attention to the thermo-fluid characteristics of the natural convection models. For that purpose, Figs. 4a–4b are used to illustrate the velocity profiles for the detailed and compact heat sink representations. The corresponding temperature profiles are given in Figs. 5a–5b. In Figs. 4a–4b, the velocity magnitudes induced by buoyancy forces can be seen to be approximately the same irrespective of the model used. In fact, the maximum velocity associated with the compact model is 0.287 m/s versus a 0.274 m/s value obtained with the detailed system. This difference is less than 5%. It may be instructive to mention that a large unducted wind

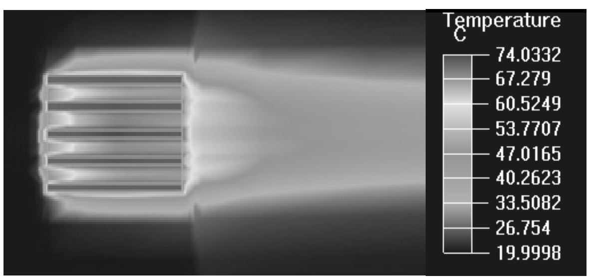


a)

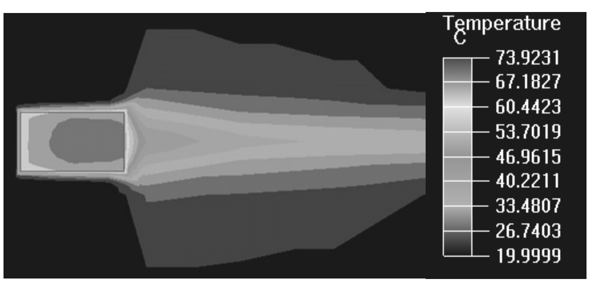


b)

Fig. 4 Velocity profiles using a) detailed and b) compact heat sink model under natural convection.



a)



b)

Fig. 5 Temperature profiles using a) detailed and b) compact heat sink models under natural convection.

tunnel was used in the natural convection case while a ducted wind tunnel was used in the forced convection case. This was done for the purpose of enabling the unrestricted spreading of the thermal and hydrodynamic plumes arising from buoyancy-driven currents.

Turning our attention to the thermal character, the heat transfer pathways shown in Figs. 5a–5b indicate that the temperature distributions around the detailed heat sinks are in good general agreement with those around the compact model. Note, in particular, the ability of the compact simulation to match, within round off error, the 74 °C maximum temperature projected by the detailed model.

2. Computational Savings

The compact heat sink representations, when used carefully, can clearly reduce the computational time and grid size without appreciable loss in accuracy. In the current study, the mesh size needed to obtain accurate thermal resistances for the detailed heat sink is about 2-10 times larger than that for the compact model. The reduction in mesh size has a significant impact on CPU run-times and resource demands. For complex fin prototypes, this gain can be substantial. Further refinements in hybrid techniques that use standard heat transfer correlations and lumped heat sink models are hence recommended. More research is also suggested to bridge the gap between existing methods and hybrid combinations similar to those described above.

V. Concluding Remarks

The analysis of heat sinks using both detailed and compact representations have been carried out for both forced convection and natural convection scenarios. The final results obtained from numerical simulations are found to be in close agreement with independently acquired experimental measurements. The two simple case studies described above confirm the usefulness and efficacy of using compact heat sink representations in concert with empirical correlations. More research is certainly needed to improve our current porous block methodology by better understanding the effects of varying exit temperatures and velocity profiles in staggered and unstaggered heat sink arrangements. Further research is also needed in characterizing more complex heat sinks exhibiting diverse shapes.

Unlike the current study where parallel-plate heat sinks are involved, there exist other cases where the choice of a characteristic length in the streamwise direction can be more challenging. When more complex body shapes are involved, it may be useful to explore using the square root of the wetted area as the characteristic length in the problem. Such a plausible choice is inspired from recent studies by Lee, Yovanovich, and Jafarpur[29] who have shown that a characteristic length based on the wetted area can lead to a more efficient and accurate representation. Such a length scale is recommended in further studies of compact models based on volumetric resistances.

In the interim, it is possible for the temperature excess used in this work to be made more sensitive by basing it on the log-mean temperature difference (LMTD) suggested by Kraus and Bar-Cohen[30] under forced convection conditions. Using the LMTD to represent ΔT may be advantageous in problems for which T_s remains constant while the local temperature of the coolant varies appreciably across the heat sink.

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