AIAA 2003-5112

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39th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit
20–23 July 2003
Huntsville, AL

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Here we describe some recent developments in analytical models that have been formulated in support of numerical and experimental studies of core gas dynamics appropriate of idealized rocket chambers. These analytical pursuits have invariably helped in providing additional physical insight while unraveling concrete parametric relations, explicit solutions, and group parameters of fundamental importance. In several practical applications, they have been invaluable in providing limiting process benchmarks needed to verify and, thereby, validate the reliability of numerical simulations. In many instances, they have enabled the specification of key similarity parameters that have been used to guide both experimental and numerical simulations. When equipped with perturbation tools, asymptotic methods have proven particularly useful in unveiling the essential features of acoustic instability mechanisms in rocket motors and other large combustors.

IT has been recently questioned whether mathematical models of rocket core dynamics may be adequate as vehicles for physical understanding, especially that burning surfaces in production motors are rough, heterogeneous, and pliable. From this perspective, a core flow that does not account for all conceivable features, including surface vibrations and complex boundary conditions, may be deemed impractical or uncertain. The purpose of the current exposition is to explore the physical benefits of mathematical models by recognizing the role, scope, objectives and recent successes of core flow idealizations. These have been motivated by important technological applications that will now be overviewed.

Being a problem of real concern in rocket motors and large gas turbines, aeroacoustic instabilities have received much scrutiny in the propulsion community. Models of these instabilities have invariably pointed to the importance of providing judicious assessments of corresponding core flow details.[1-6] The desire for explicit flow models has also been motivated by the need to physically understand the intricate coupling between unsteady pressure waves and gas motions.[7-10] Being ultimately suggested by repeated tests, not only does this inevitable pairing provoke unsteady burning, but also generate intense sound-pressure levels and boundary-driven vortices.

This delicate interplay of underpinning core flow mechanisms has inspired over the years several capable theoreticians to seek physical idealizations. The goal has been generally set to carefully isolate the intricate mechanisms by parametric linearization or vector decomposition. In the midst of this unusually complex problem, the quest for basic answers has often become a central focus.

Pioneered by Culick[11-17] and Flandro,[1-6] theoretical studies have elucidated a number of physical features in rocket motors. Among them were the multi-dimensional spatial and temporal velocity, vorticity, and stress distributions along the length of simulated motors.[18] Points exhibiting maximum and minimum stress disturbances were identified while the acoustic character in the chamber was being disclosed.[19-25] These analyses have been accomplished by first unraveling the problem's principal convection-diffusion equations and their underlying multi-scale structure. The latter exhibited nonlinear characteristic lengths whose resolution required a separate mathematical treatment.

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The presence of dissimilar scales in the resulting Navier-Stokes equations[26] has spurred on the development of two mathematical procedures appropriate of a class of boundary-value problems with nonlinear scales. The first was a variant of the derivative-expansion method and was dubbed, for want of a better name, the composite-scaling technique (CST).[27,28] The second was a more general, space-reductive approach based on Prandtl's principle of least singular behavior. The second was coined the generalized-scaling technique (GST) because of its ability to preclude guesswork in the identification of inner coordinate transformations.[19-22] Its outcome coincided with the traditional WKBJ solution for problems in which both methods were suitable. In view of the wide applicability of the convection-diffusion equations precipitated by core flow models, the presentation of CST and GST space-reductive techniques became an important result in its own right.

As suitably put by von Kármán,[29] "it was the simple but powerful mathematical statement, preferably a differential one, that held the key to the whole complicated affair." In this spirit, having realized differentially accurate approximations, implications on core acoustics could be diligently pursued.[4,5] In the process, new destabilizing sources of energy could be uncovered mainly due to interactions with the transpiring surfaces.[10] In actuality, incorporation of these additional sources by Flandro have markedly improved our predictive capability prior to motor development.[4,5] As shown in his 1995 paper,[5] a favorable agreement with laboratory measurements could be obtained when vortical corrections were included. In the same context, the embodiment of unsteady rotational effects at the forefront of the energy balance equations could actually give rise to several new energy-related terms that were discounted previously.[30] These corrective terms could explain, in part, the presence of parietal vortex coupling[31,32] and turbulent interactions with the mean and unsteady flow components. While analytical solutions were sought for simple geometric shapes, a complete integral formulation of the stability integrals was provided by Chibli, Majdalani, and Flandro[33] to analyze realistic solid rocket motors with arbitrary grain configurations.

While the roadmap for flame zone analysis was being sketched by Flandro,[6] approximate asymptotic solutions for the temperature contours within the motor were under development.[34] Despite their intrinsic simplicity (reflected in the use of a 'super-heat source'), the asymptotically-derived analytical solutions could be calibrated to emulate isotherms acquired from detailed numerical simulations.[34] The key resided in the heat source attributes that could mimic the heat of reaction above the burning surface. The reader may find particularly interesting the recent numerical simulations of the burning zone by Chu, Yang, and Majdalani.[35] These simulations have confirmed the presence of a virtually non-reactive, isothermal region above the flame where analytical approximations, such as the ones under scrutiny, become very effective in describing the gas dynamics. Since the outer region constitutes 98% of the chamber volume, the practicality of analytical solutions is evident.

A departure from the ideal model can, of course, be expected due to external vibration, flight acceleration, particle debouching and non-uniform injection velocity at the chamber walls. However, as these mechanisms occur at random frequencies, the acoustic disturbances that they produce only help to excite the natural frequencies of the chamber; by so doing, they serve to sustain the source of acoustic waves that is already assumed to exist in theoretical models. It should also be noted that these studies are based on velocity boundary conditions at the porous walls. They are not concerned with the level of external pressure that is required to impart a given injection velocity. The porosity of the walls becomes immaterial so long as the incoming flow is sufficiently uniform. Nevertheless, porosity does not elude theoretical idealizations. It is recognized and described by Zhou and Majdalani[36] as the agent in control of the wall injection coefficient. As for the oscillating pressure forces, only in the case of resonant burning does the fluid-structure interaction become a concern. In other instances, this coupling may be safely ignored. Furthermore, insofar as rockets operate under choked nozzle conditions, the influence of external noise propagating back into the chamber is insignificant. The isobaric outlet conditions selected in some theoretical models are used to mimic nozzeless motor simulations conducted in controlled laboratory environments.

In support of theoretical approximations, equally appreciable laminar and turbulent segments have been identified in a series of studies conducted by Apte and Yang,[37,38] Liou and Lien,[39] and Beddini and Roberts.[40,41] These teams have simultaneously concurred that turbulent simulations in the downstream sections of long motors tended to exhibit convincing similarities with corresponding basic laminar flow results. In view of this important realization, the unsteady flow solutions derived by Flandro[5] or Majdalani and Van Moorhem[8] have been used to approximate the basic deterministic features observed in turbulent flow results.[42,43] Additionally, they have been used to provide accurate predictions along the forward half of the motor where laminar conditions are known to prevail.

Aside from these sequential contributions, it must be noted that recent efforts have been successful in producing analytical core flow solutions that are sensitive to the movement of transpiring boundaries. The reader is referred, in that regard, to the planar and axisymmetric solutions presented by Zhou and Majdalani[36,44] or Majdalani, Vyas, and Flandro.[45,46] Therein, wall regression is accounted for alongside viscous and rotational features. By applying a mass balance across the receding interface, the dependence of thrust on propellant morphology and density variation is unraveled. Work is currently underway to simulate motor chambers with arbitrary taper.[47] It is hoped that these incremental advancements in theoretical models will bring us closer to an idealized motor with varying cross section and grain shape. Incorporation of swirling effects has also been accomplished in a forthcoming report by Vyas and Majdalani. Analytical solutions have been useful in providing limiting-process case-studies against which detailed numerical simulations could be immediately compared.[48]

Regarding the role of mathematical solutions in gaining deeper physical insight, the technical examples recounted above may perhaps suffice. According to Sears,[29] it was always "the fundamental differential relationship, when it succinctly described the important phenomenon, that von Kármán always sought." For what could outperform analytical tools in exposing the basic interdependencies in a problem? Consider the Shvab–Zeldovich equations, for example. Despite their unrealistically simple sets of assumptions, they remain perhaps the most pivotal in elucidating the physics of combustion in standard surveys.[49] According to von Kármán,[29] that was "the best of applied mathematics."

The mathematical models described earlier can never be substituted for multi-stage, multi-module, fully coupled numerical simulations that could be supported by extensive experimentation. Such computational capabilities are currently under development by a number of dedicated scientists, including Dick, Heath, Balachandar, Moser, Fiedler, Najjar and others at the Center for Simulation of Advanced Rockets.[50-53] There is no doubt that the efforts expended by this body of scientists stand to provide the best diagnostic and predictive tools in the history of rocket development. However, their efforts are not without challenges either.

According to Dick, Heath, and Fiedler,[50] their efforts require the integration of "approximately thirty-five faculty members, thirty research scientists and programmers, and forty graduate students from a dozen academic departments." To provide meaningful data, the integrated modules have to be executed in parallel on 512 processors on a CrayT3E. Discretization requirements are obviously enormous, encompassing millions of fluid cells and several hundred thousand structural elements. Therein, the propellant surface is assumed to ignite everywhere at the beginning of the simulation. However, "regression of the propellant due to burning [is] neglected because of the short physical time reached in the simulation, 0.1 sec after 10 days of wall clock time." Based on their experience with the integrated code, Dick, Heath, and Fiedler[50] remark that "a high fidelity simulation of the RSRM [Reusable Solid Rocket Motor] over the entire two minute burn time would require a prohibitive amount of execution time, even on the largest and fastest platforms available now or in the near future." Rather than allow these challenges to deter their activities, one must acknowledge that the CSAR scientists have chosen the course of continual improvement in their quest for a systemic code. In the absence of suitable experimental data for validation purposes, these investigators[54] have resorted to expeditious, limiting process verifications based on the approximate solution obtained by Majdalani and Van Moorhem.[8] This solution is identical in scope to the one under scrutiny. For the fundamental and first harmonic oscillations modes that dominate in acoustic instability assessment, analytical solutions and computational results obtained at CSAR were virtually indistinguishable.

In order to justify the modern need to pursue an analytical side to a full-scale investigation, a plethora of convincing arguments may be offered. One example can be derived from a propulsion related study of international repute. In investigating aeroacoustic technologies to reduce the excess noise of Concorde jet engines during take-off and landing, the use of applied mathematics by Crighton and Williams[55,56] has proven invaluable to the comprehensive Anglo-French research endeavor. Despite the obvious dissimilarities between turbulent shear layers surrounding the propulsive Rolls Royce jets and the simplistic laminar models based on acoustic analogies, the use of powerful mathematical techniques has unraveled the means to suppress the ensuing sound and vibration. By seeding the jet with sound upstream of the nozzle, a coherent turbulent structure could be promoted in a manner to reduce the jet noise without impairing thrust and the aircraft's ability to climb. Thus, by deliberately superimposing an aeroacoustic source whose troughs coincided with the pressure peaks of an existing acoustic environment, a quieter combination could be precipitated. The degree of suppression obtained in this way depended, of course, on the accuracy of the active cancellation models which were suggested analytically, and then refined via numerical and experimental simulations. Similar analyses have recently provided multiple-scale solutions for modal sound transmission through turbofan aircraft engines with both hard and acoustically treated inlet walls.[57,58] These solutions by Rienstra and co-workers[57,58] have closely agreed with output data gathered from finite-element codes.

Regarding the ability to provide more realistic models, it should be noted that efforts in this direction are constantly underway. Examples include current analytical abilities to account for wall regression, unsteady vorticity, viscous diffusion, bore taper, varying cross sections, and non-uniform injection. In the past, propellant morphology and heterogeneity were shown, however, to be more relevant to the physico-chemistry of combustion than to the acoustic environment within the chamber. As a matter of record, these aspects are treated separately outside the core flow module in the detailed simulations by CSAR. In fact, despite our earnest desire to account for surface vibrations and motor irregularities (due to inhibitors, igniters, submerged nozzles, interface gaps, O-rings, conocyls and slots,[59]) the added complexities that accompany these geometric disparities need to be handled piece at a time. Nonetheless, approximate solutions of the type we have described remain fairly useful in relaying the basic physical aspects of the problem while helping to validate more elaborate numerical simulations. In the case of wall regression that was particularly pointed out, recent findings have indicated its small influence on the overall acoustic field.[36,44,45] This could perhaps justify its dismissal in many laboratory, computational, and theoretical investigations.

In the recent CSAR simulations by Venugopal, Najjar, and Moser,[52] the rocket motor is idealized as a plane channel with wall injection across two smooth porous walls separated by a distance of 2δ. This configuration resembles the one adopted by the French scientists in their VECLA apparatus (*Veine d'Études de la Couche Limite Acoustique*), and the geometry used in the article under consideration.[60] Nonetheless, in order to homogenize the Navier-Stokes equations in the streamwise direction, these researchers have assumed that turbulent fluctuations vary over a length scale of $\mathcal{O}(1)$, and that turbulent amplitudes grow over a larger length scale of $\mathcal{O}(1/\varepsilon)$. To take into account the non-uniform variations over these two rational length scales, slow and fast variables are introduced via $x_s = \varepsilon x$ and $x_f = x$. A two-variable multiple-scale expansion is then carried out before numerical simulations could be broken into a number of discrete solutions at user-prescribed stations –namely, one for each value of ε.

It should be noted that, throughout this two-scale analysis, the perturbation parameter is taken to be $\varepsilon = \delta / x$, where x is measured from the head end of the simulated rocket. Since δ represents the half-height of the porous channel, each streamwise location in the rocket motor could be identified with a particular value of ε. Specifically, the primary perturbation parameter ε appears as a variable whereas the slow scale $x_s = \varepsilon x\,(= \delta = \text{const})$ is used to represent an artificial coordinate. Venugopal, Najjar, and Moser[52] later indicate that, since their analysis assumes ε to be small, their model will "be strictly valid only at streamwise locations close to the nozzle of the rocket motor." Being applicable to downstream motor sections, their work complements the basic relations derived by Flandro[5] and Majdalani and Van Moorhem.[18] The latter provide closed-form solutions that are valid everywhere except near the nozzle where turbulence and compressibility effects become eminent. It should also be emphasized that, based on the CSAR simulations,[52] the mean flow is found to be approximately isobaric, a feature that has been exploited to linearize the pressure response as part of the analytical framework.[18]

Regarding the elements of uncertainty and practicality in mathematical models, these ailments are also of much concern in both computational and experimental procedures. The test of validity is always whether or not predictions represent adequate approximations to the problem under consideration. The reader is invited to consult with Majdalani and Flandro[21] wherein theoretical solutions are shown to agree very closely with computational data acquired totally independently by Roh and Culick.[61] In the same study, asymptotics are shown to provide fair predictions of experimental measurements.

Regarding the element of uncertainty in propellant composition, the most illuminating discussion could perhaps be borrowed from Buckmaster, Jackson, and Ulrich.[62] Following a terrific simulation of time-accurate propellant burning, these researchers quite elegantly state in their 2001 conclusions: "The specific model that we have used here is a simple one, indeed over-simple, and one that omits much physics. In a word, it is false. But unless we have made numerical errors, our results are not wrong. Indeed, 'right' and 'wrong' is not the correct dichotomy within which to evaluate the results. What is important is the extent to which they imitate the burning of real propellants, and there are things to be learnt from both its failures and its successes in this respect. We anticipate an evolutionary process in which new ingredients will be added, and existing ingredients modified, and we are confident that there will be useful successes despite the fact that for such a complex problem the model will be *eternally false*."

These realistic statements seem to apply equally well to mathematical models under development. As alluded to by Buckmaster, Jackson, and Ulrich,[62] current pursuits may be likened to the efforts of alchemists in 18th century Europe. Despite their confrontation with numerous challenges and hopelessly complicated successions of obscure trials, alchemists never doubted that a relative simplicity lurked behind the apparent complexities (e.g., Culick's mean flow profile[12]). Thus, in their efforts to transmute copper into gold, they discovered a large number of useful substances including ammoniac, alcohol, and mineral acids on the basis of which modern chemistry and rocketry would later be founded. As for the crisp analytical formulations debated earlier, the main point in obtaining them is, perhaps, to thwart the claim by alchemists that all true knowledge is to be repeatedly found and lost.

Acknowledgments

The first author is thankful for NASA and the Wisconsin Space Grant Consortium for their 2002-2003 Research Infrastructure and Higher Education Seed Grants.

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